Quantenfeldtheorie

… in 90 Minuten

Robert Harlander

RWTH Aachen University

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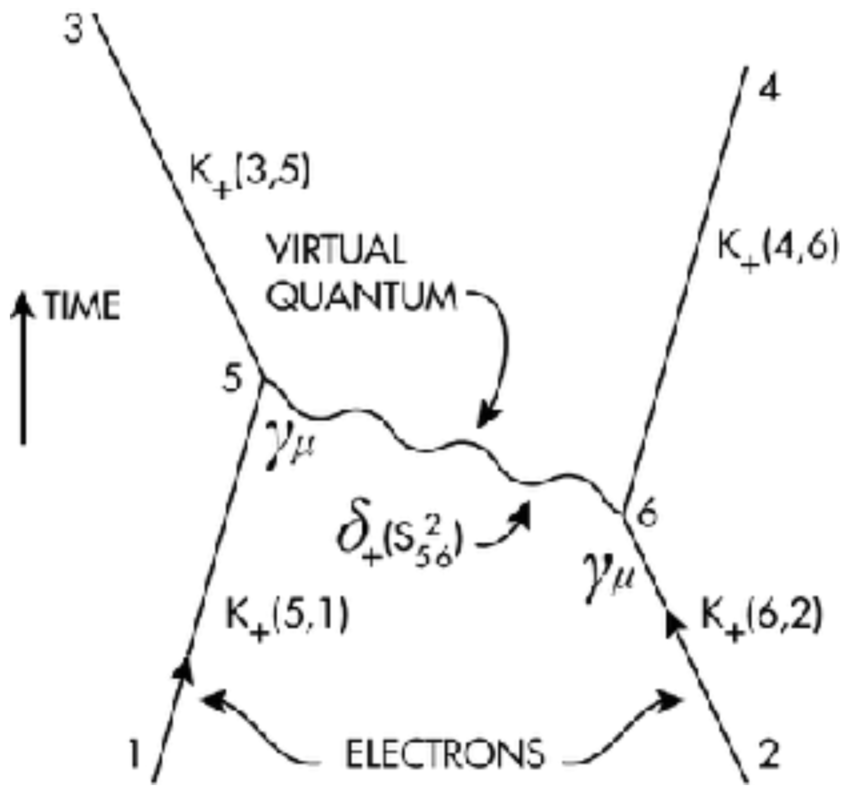
Ziele



Feynman ’49

- Wo kommt das her?
- Was kann man damit machen?
- Wie kann man das interpretieren?

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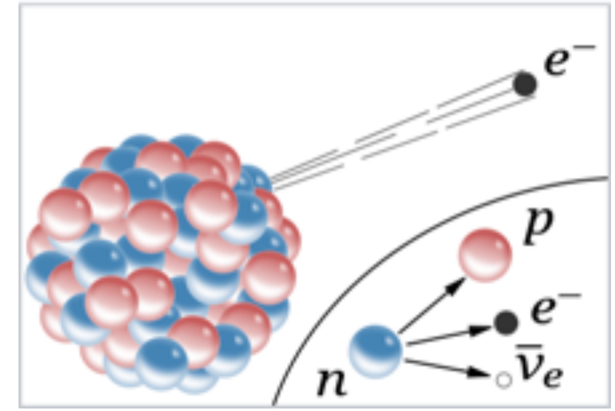
Ziele

β-Zerfall: $n \to p + e^- + \bar{\nu}_e$

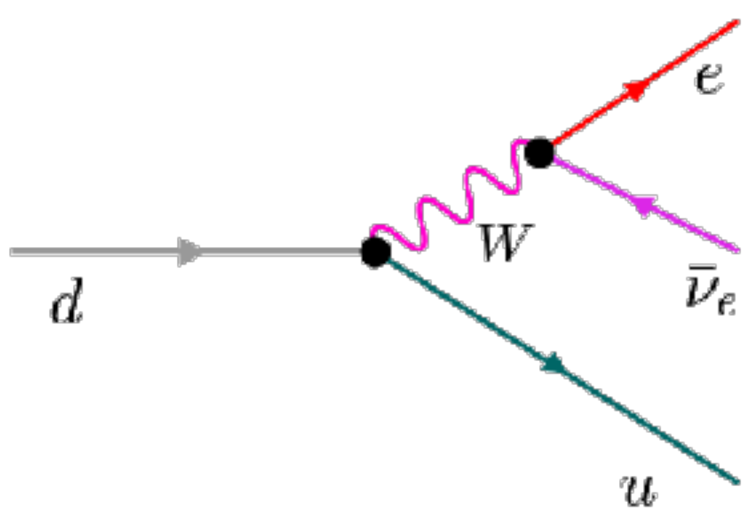
$n = (ddu)$ $p = (uud)$

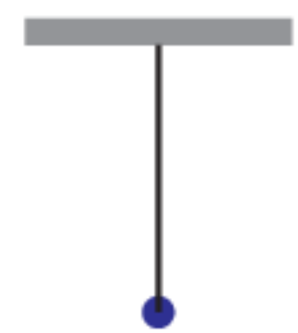


$${}^{198}_{79}\mathrm{Au} \to {}^{198}_{80}\mathrm{Hg} + \mathrm{e}^- + \bar{\nu}_e$$

β-Zerfall: $d \to u + e^- + \bar{\nu}_e$

wohin verschwindet das d ?
woher kommen $u, e^-, \bar{\nu}_e$?





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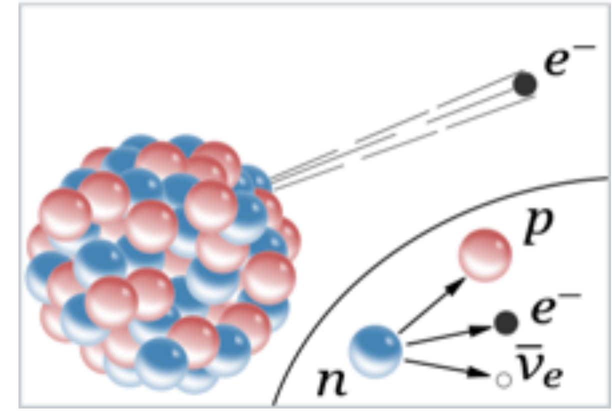

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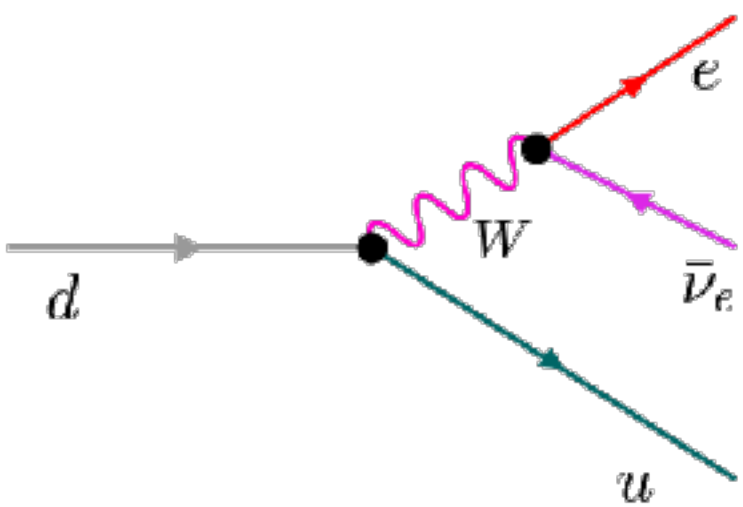

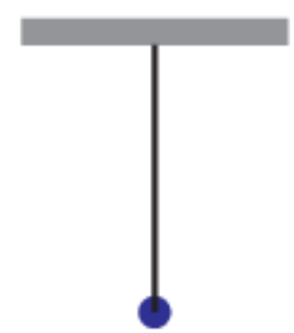

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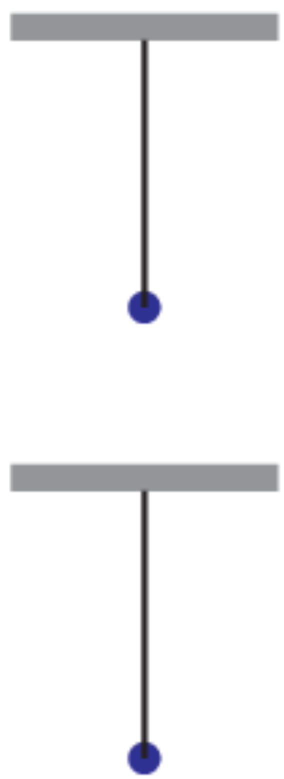
z.B. Photon = “Lichtteilchen”

klassisch: Licht = elektromagnetische Welle

Quantenphysik: Photon = minimale “Oszillation” der Welle

klassische Mechanik



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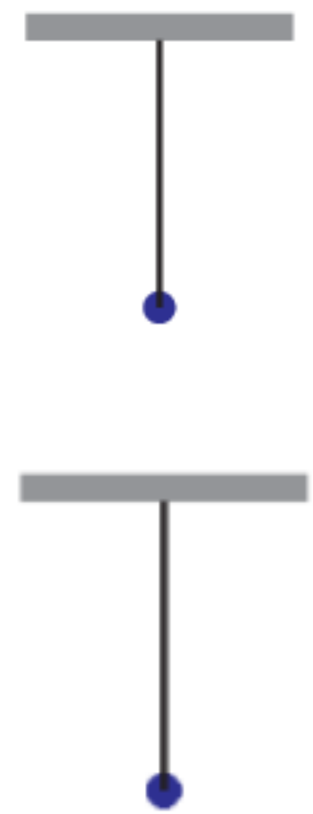

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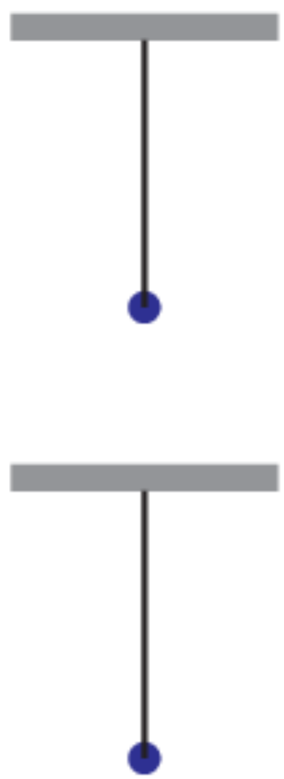

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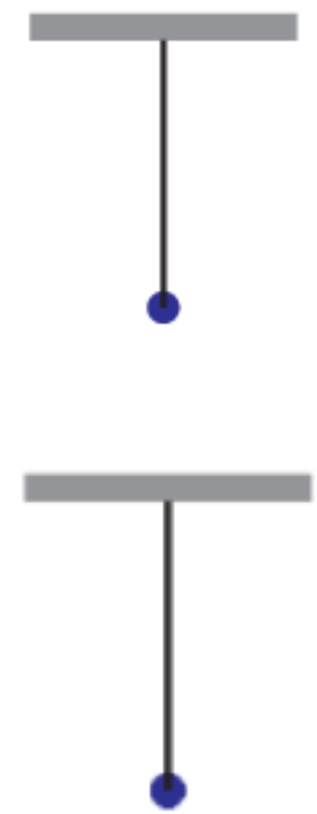

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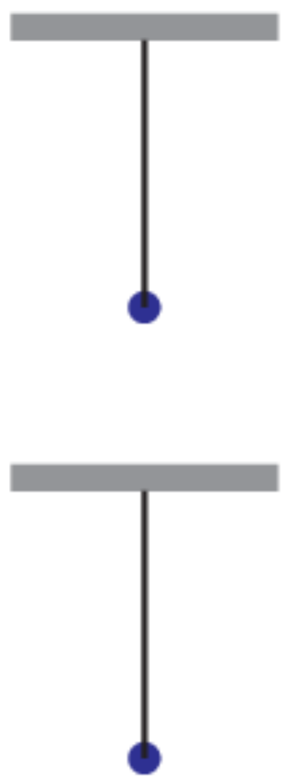

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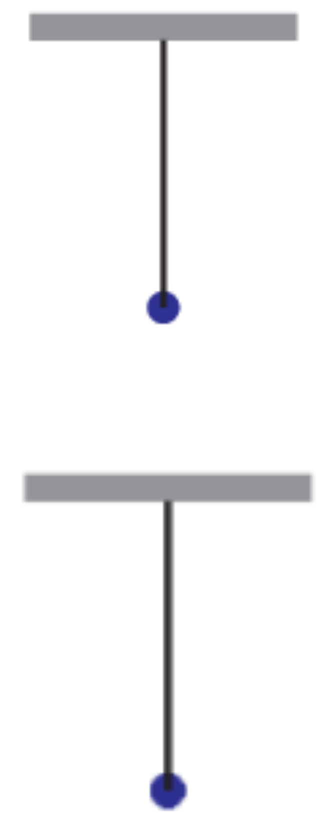

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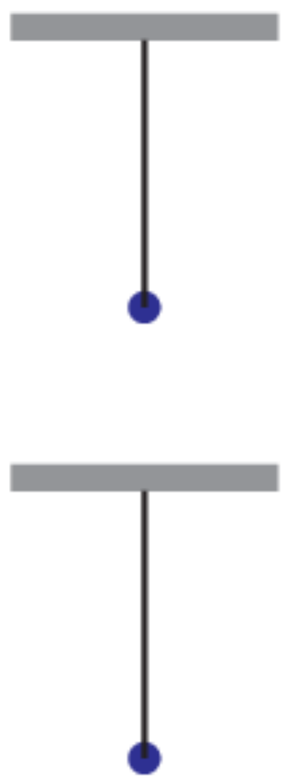

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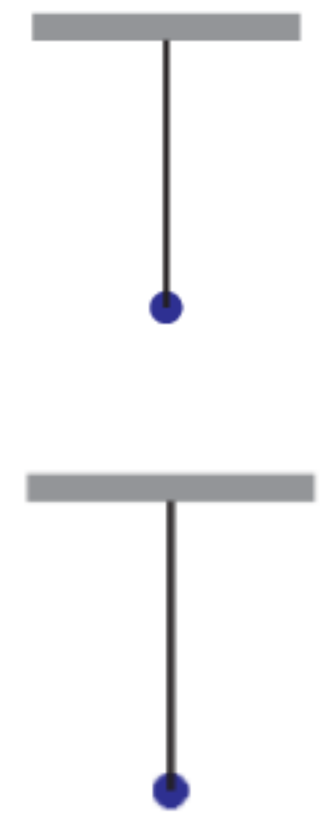

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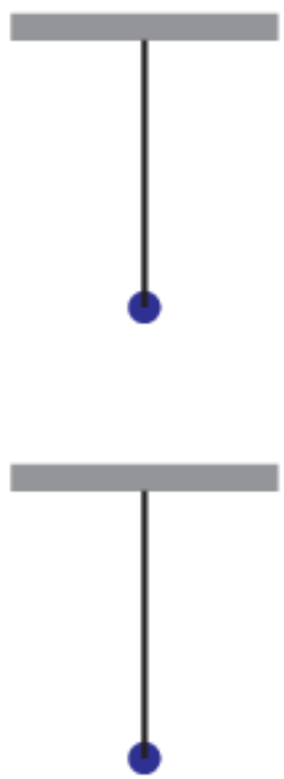

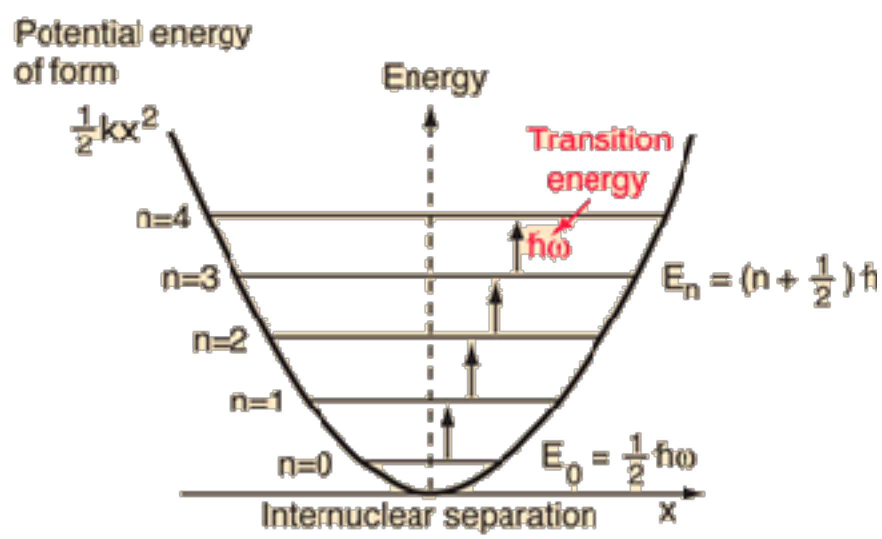

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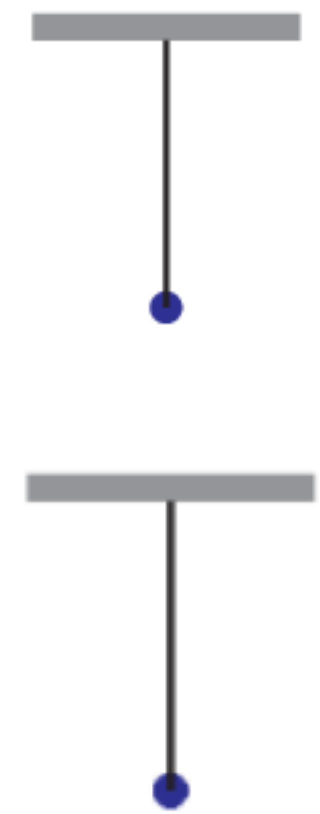

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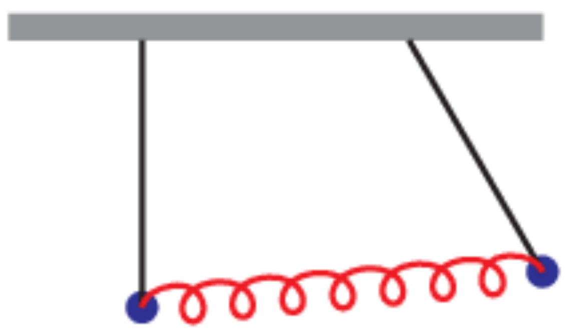

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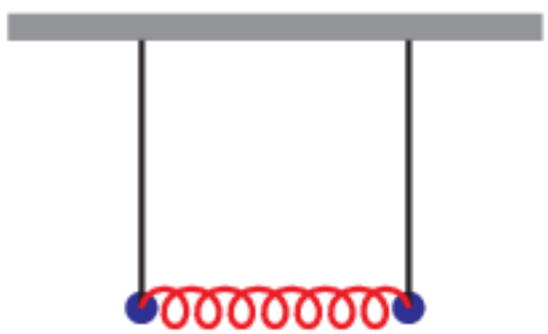

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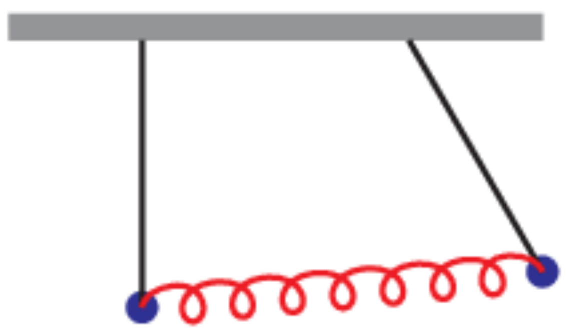

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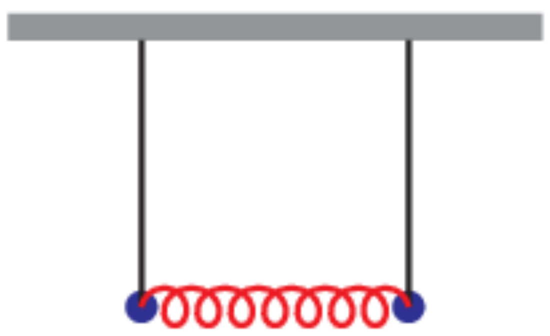

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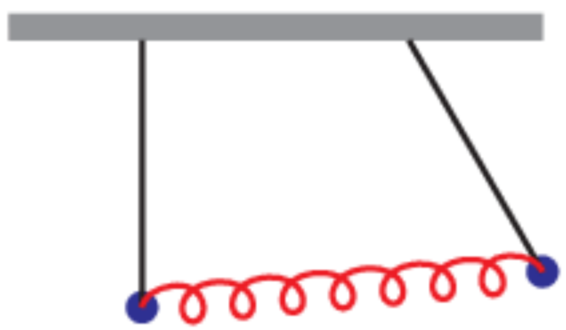

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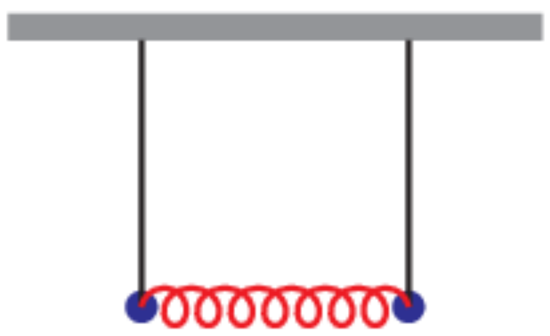

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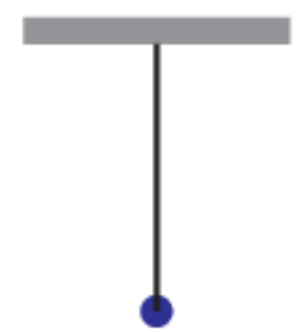
Zentrales Objekt in der Physik: **Lagrangefunktion**

$$L(\dot{q}(t), q(t))$$

Enthält die gesamte Information über das System:
Symmetrien, Erhaltungssätze, Zeitentwicklung,…

z.B. Fadenpendel:



$$L = \frac{m\,\dot{\varphi}^2}{2} + a\,\varphi^2 + b\,\varphi^4 + c\,\varphi^6 + \ldots$$

$a,\ b,\ c,\ \ldots$ müssen experimentell bestimmt werden

für kleine Winkel: $$L = \frac{m\,\dot{\varphi}^2}{2} - \frac{m\omega^2}{2}\varphi^2$$

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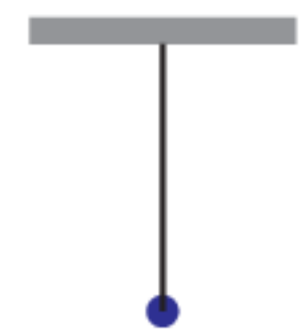

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Zentrales Objekt in der Physik: **Lagrangefunktion**

$$L(\dot{q}(t), q(t))$$

klassische Bewegungsgleichung folgt aus

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0$$

Euler-Lagrange-Gleichungen

$$L = \frac{m\,\dot{\varphi}^2}{2} - \frac{m\omega^2}{2}\varphi^2 \qquad \Rightarrow \qquad \ddot{\varphi}(t) - \omega^2\varphi(t) = 0$$

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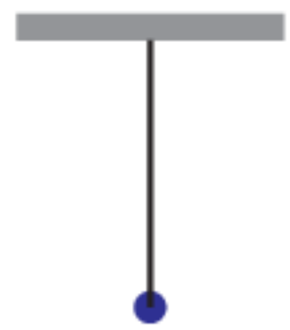

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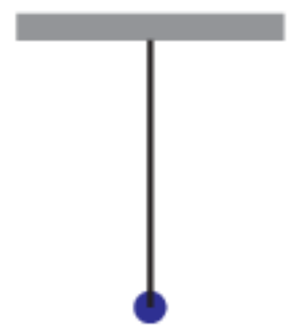

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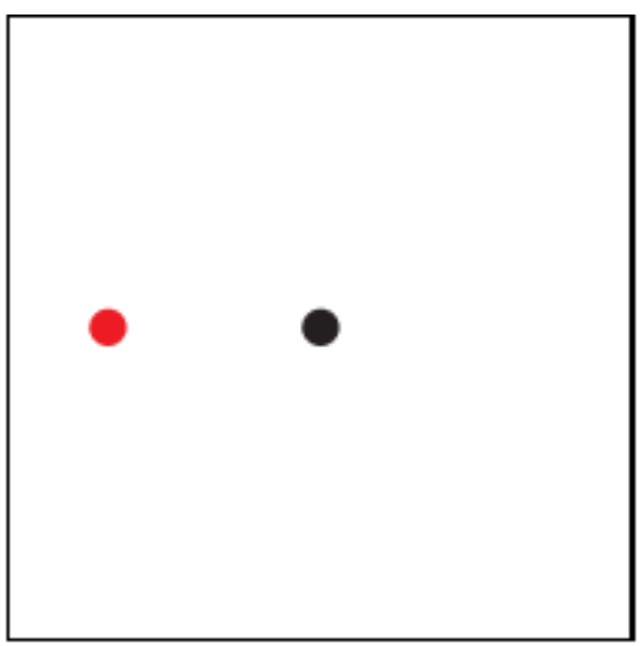
klassische Physik

$$m\ddot{\mathbf{x}} = \mathbf{F}$$

Anfangsbedingungen: $\{\mathbf{x}(t_0),\ \mathbf{v}(t_0)\}$

“Zustand”: $\{\mathbf{x}(t),\ \mathbf{p}(t)\}$

$\Rightarrow$ Bahnkurve $\mathbf{x}(t),\ t = [t_a, t_e]$



Quantenphysik

$$(\Delta\mathbf{x})(\Delta\mathbf{p}) \geq \frac{\hbar}{2}$$

?

Zustand: $|\psi(t)\rangle$

Operatoren: $\hat{\mathbf{x}},\ \hat{\mathbf{p}},\ \hat{\mathbf{L}},\ \hat{\mathbf{H}},\ \ldots$

i.A.: $\hat{\mathbf{H}}\,|\psi(t)\rangle = |\chi(t)\rangle$

“Eigenzustände”:

$$\hat{\mathbf{H}}\,|\psi_E(t)\rangle = E\,|\psi_E(t)\rangle$$

$$\hat{\mathbf{x}}(t)\,|\psi_x(t)\rangle = x\,|\psi_x(t)\rangle$$

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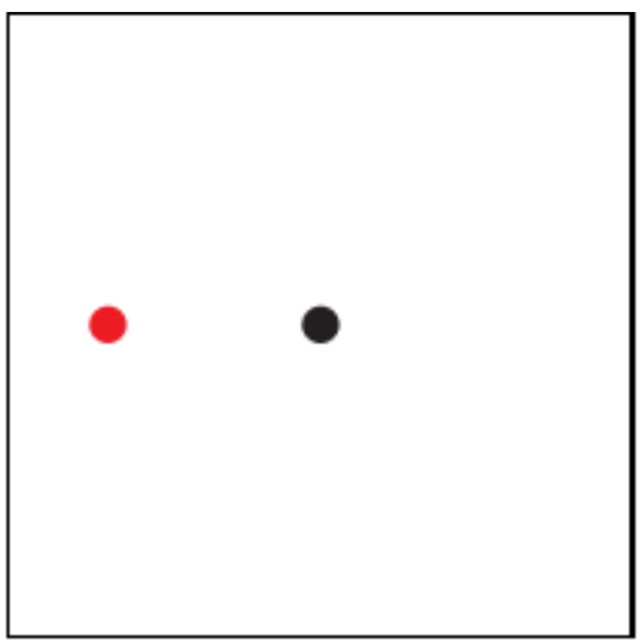

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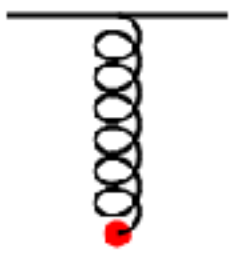
$$p = \frac{\partial L(x,\dot{x})}{\partial \dot{x}} = m\dot{x}$$

$$[\hat{x},\hat{p}] = i\hbar$$



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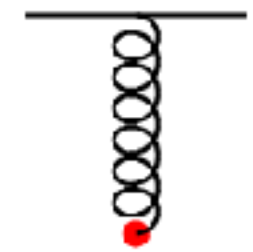
Schrödinger-Gleichung:

$$i\frac{\partial}{\partial t}\psi(x,t) = H\psi(x,t)$$

$$H = p\dot{x} - L$$



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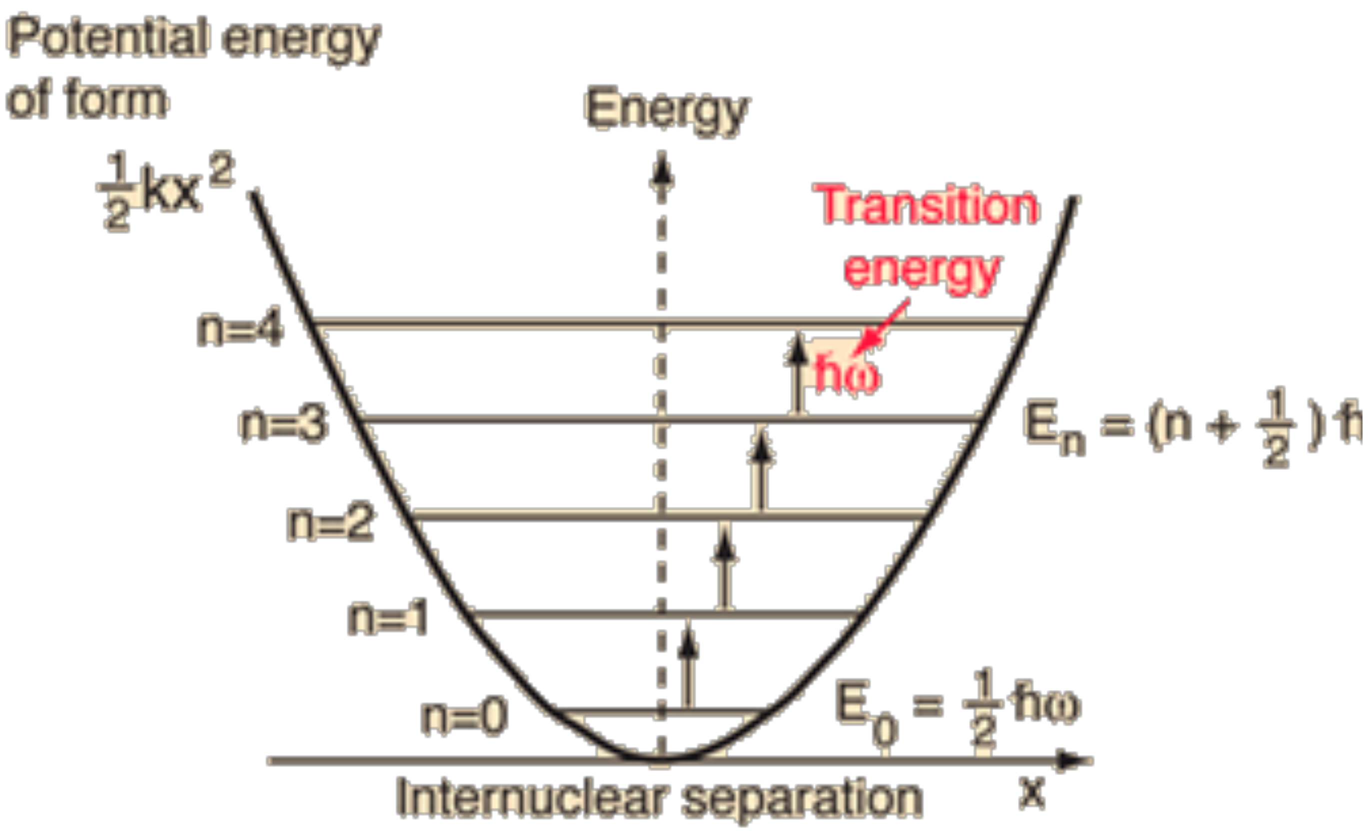

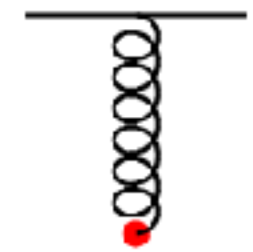

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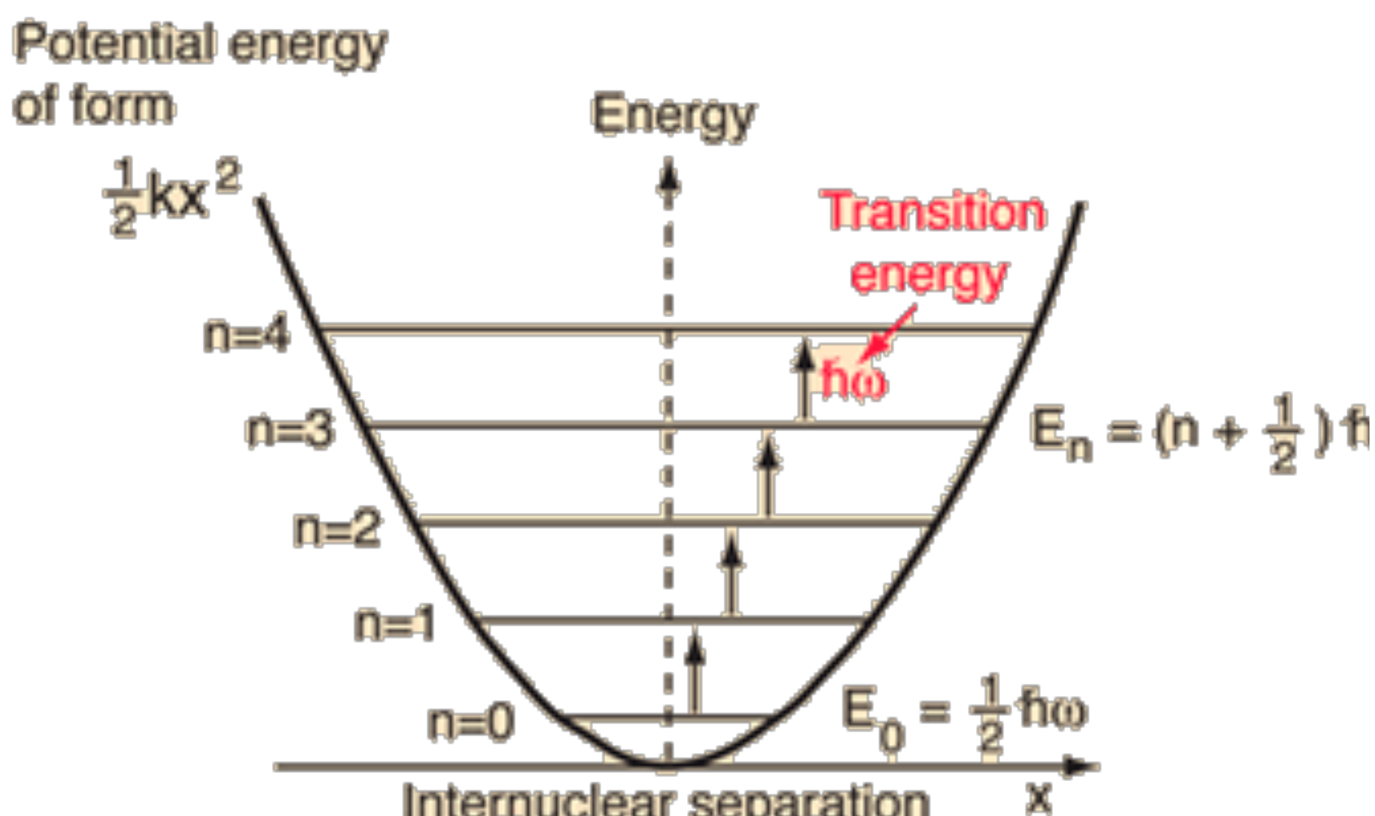

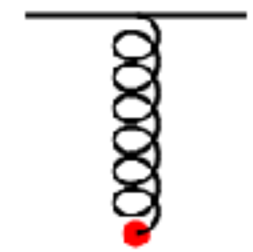

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$$\hat{\mathbf{p}}(t)\,|\psi_x(t)\rangle \sim \int dx\,|\psi_x(t)\rangle$$

$$\hat{\mathbf{x}}\,\hat{\mathbf{p}}\,|\psi\rangle \neq \hat{\mathbf{p}}\,\hat{\mathbf{x}}\,|\psi\rangle$$

$$\hat{\mathbf{x}}\,\hat{\mathbf{p}}\,|\psi\rangle = \hat{\mathbf{p}}\,\hat{\mathbf{x}}\,|\psi\rangle + i\hbar\,|\psi\rangle$$

$$\hat{\mathbf{x}}\,\hat{\mathbf{p}} - \hat{\mathbf{x}}\,\hat{\mathbf{p}} = i\hbar$$

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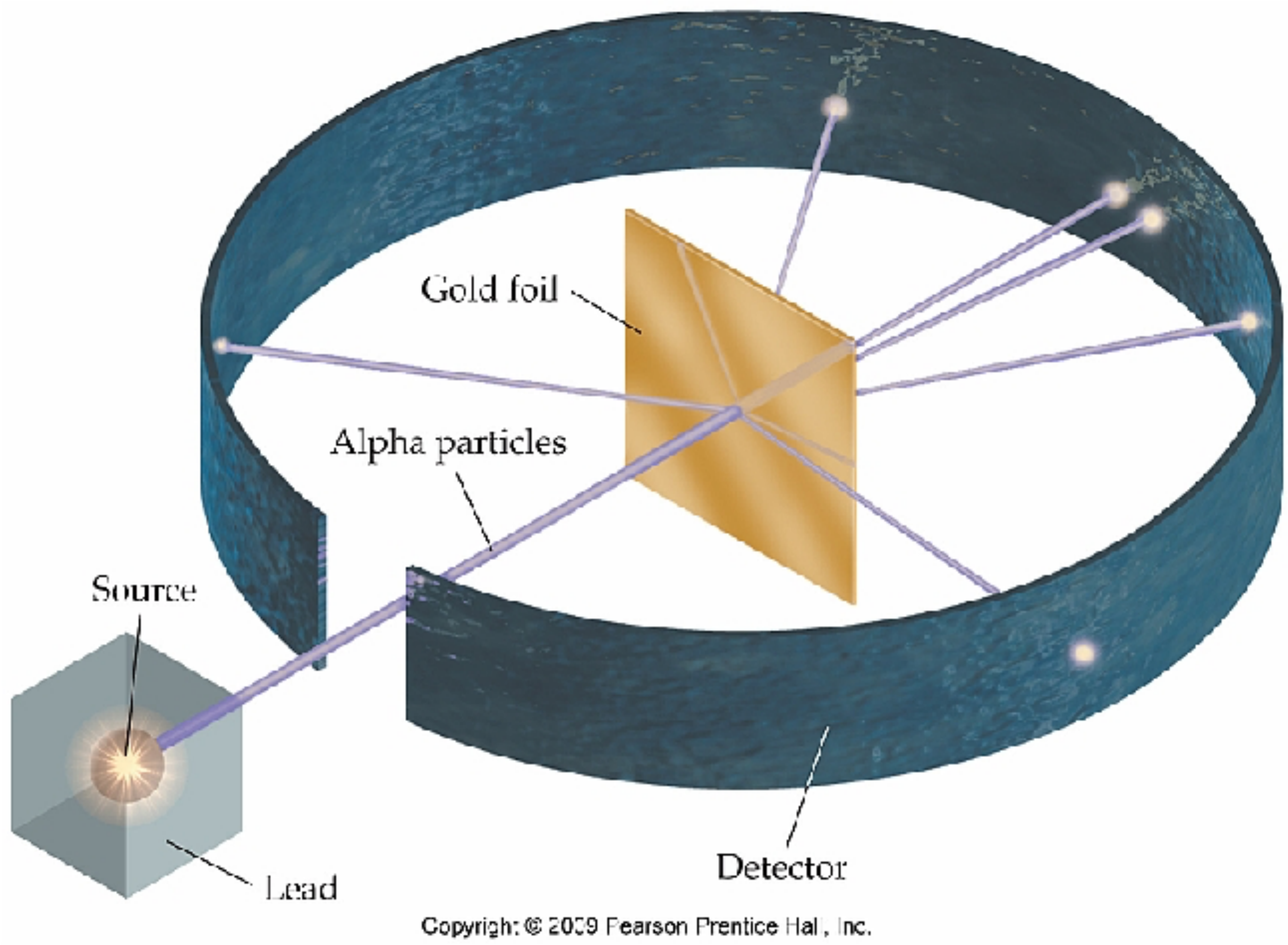

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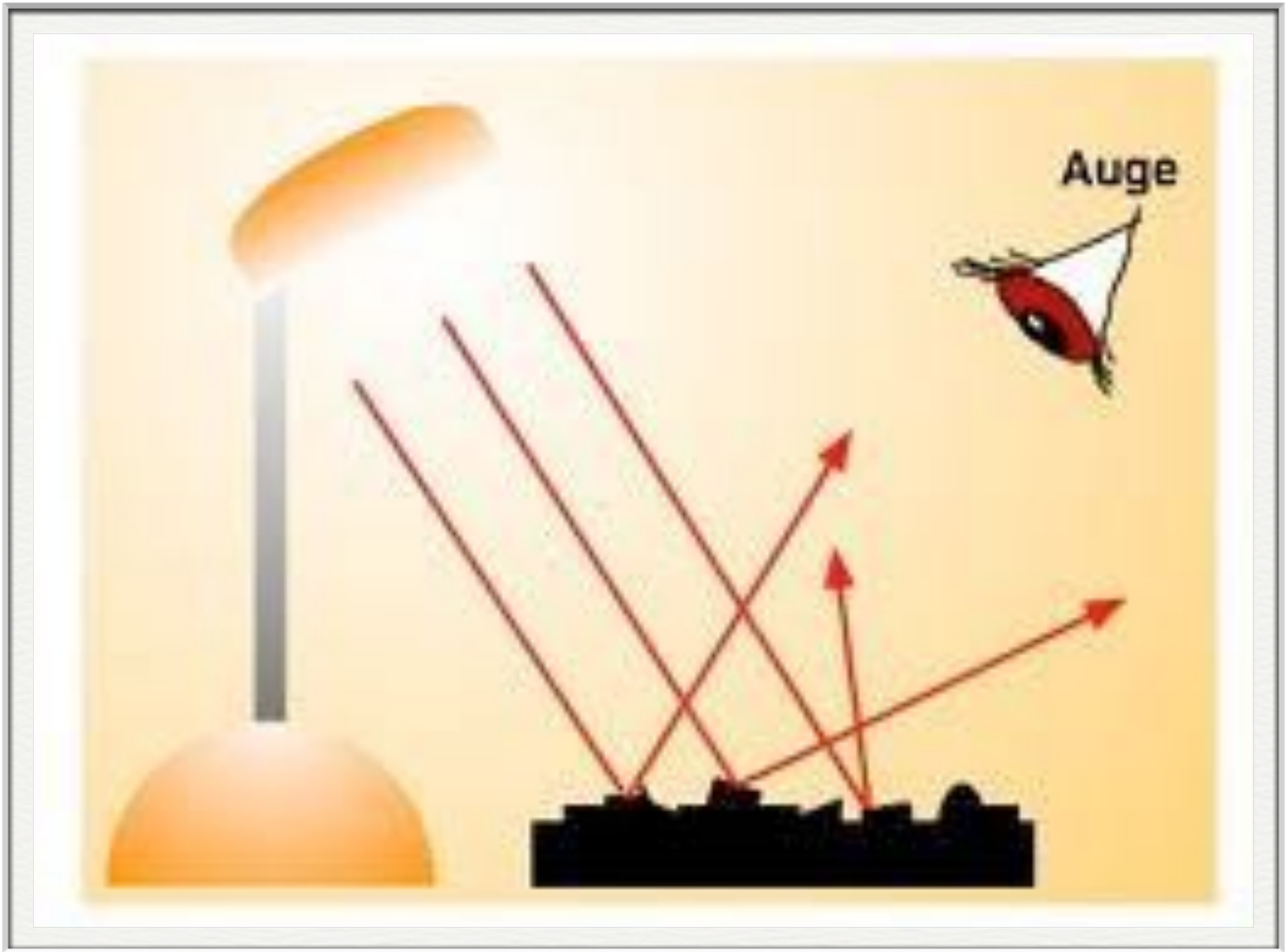

Streuexperiment

z.B. Elektron + Positron → Photon + Photon

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$$e^+e^- \rightarrow \gamma\gamma$$

Streuexperiment:

- Wahrscheinlichkeit?
- Photon in Richtung α?
- … ?

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Gegeben: Anfangszustand $(t \to -\infty)$

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Streuexperiment:

- Wahrscheinlichkeit?
- Photon in Richtung α?
- … ?

Gegeben: Anfangszustand $(t \to -\infty)$

Gesucht: Wahrscheinlichkeit für Endzustand $(t \to \infty)$

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Zentrales Objekt in der Physik: **Lagrangefunktion**

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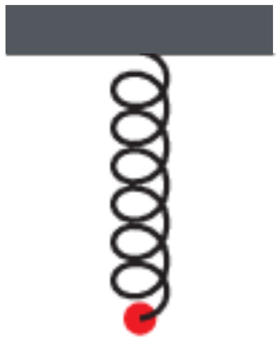

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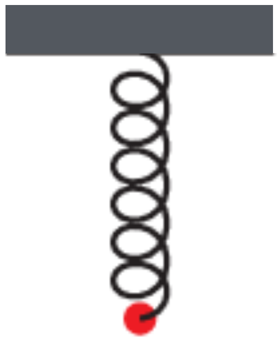

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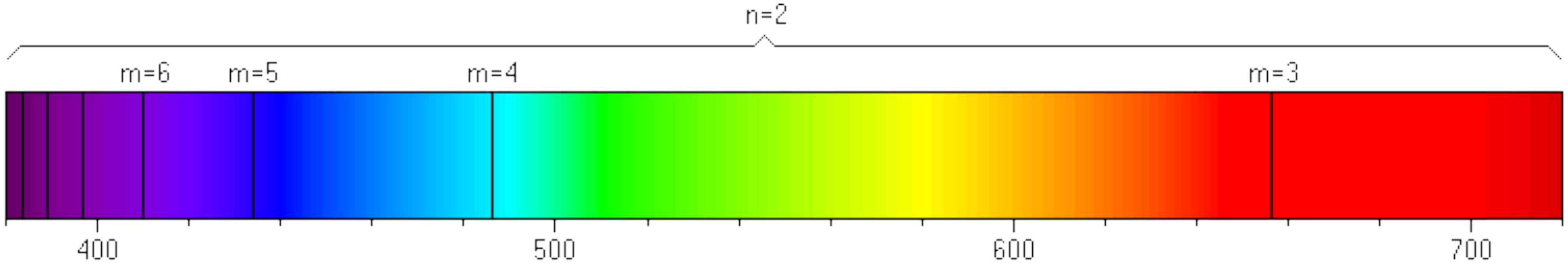
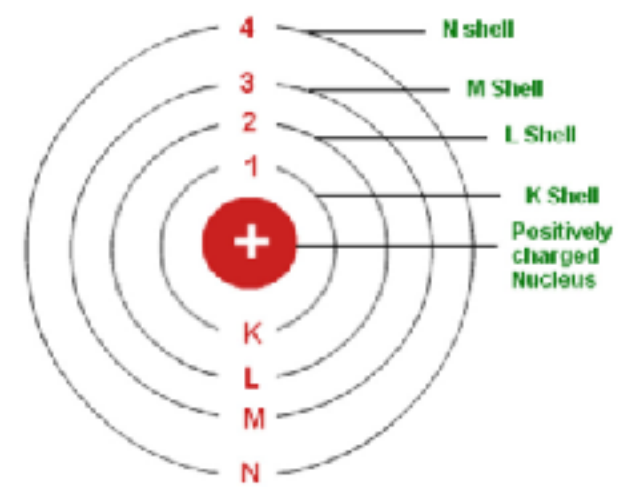
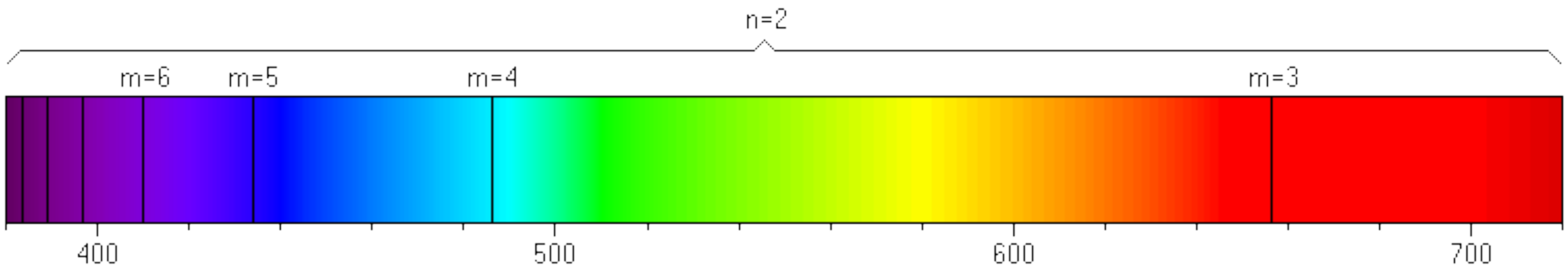


Die Natur ist quantisiert



z.B: Balmer-Serie

$$E_n = -\frac{13.6}{n^2}\,\text{eV}$$



Die Natur ist quantisiert

z.B: Balmer-Serie

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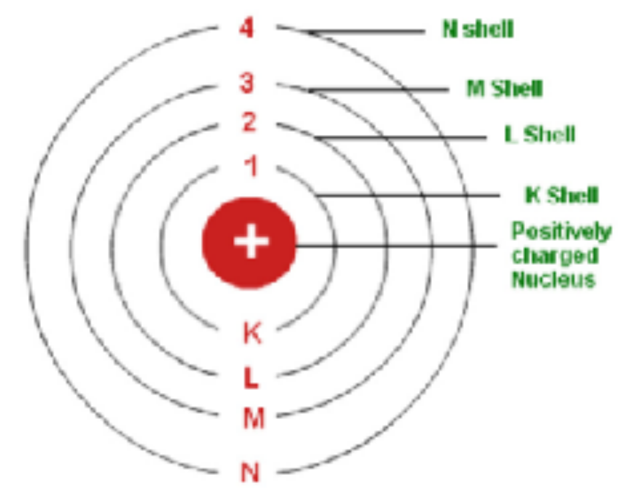
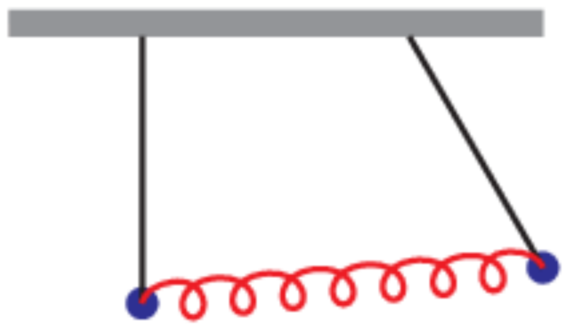


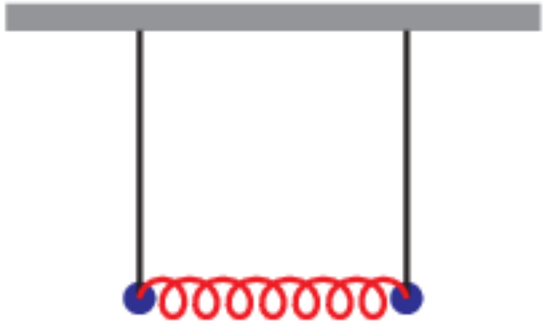
Quantenmechanik: klassische Lagrangefunktion, aber:

$$[x, p] \equiv xp - px = i\hbar$$

wobei $p = \frac{\partial L(x, \dot{x})}{\partial \dot{x}}$ konjugierter Impuls

cf. classical
exchange of energy:





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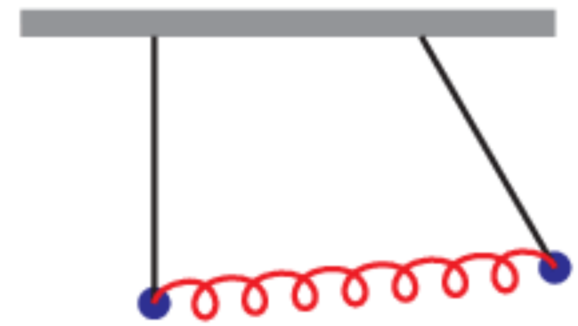

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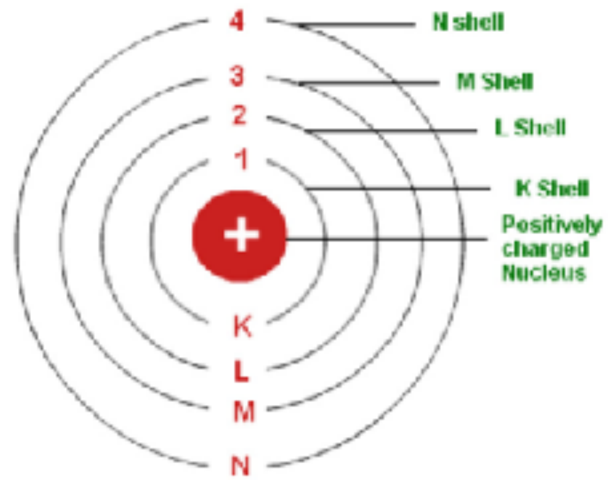

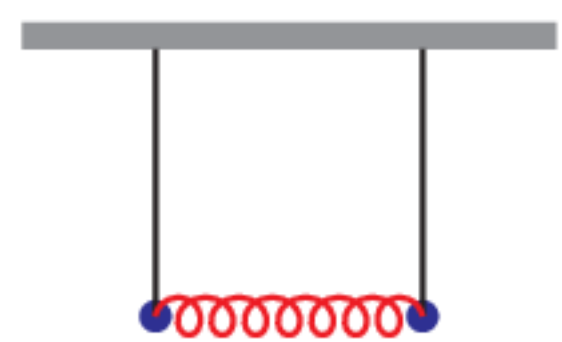

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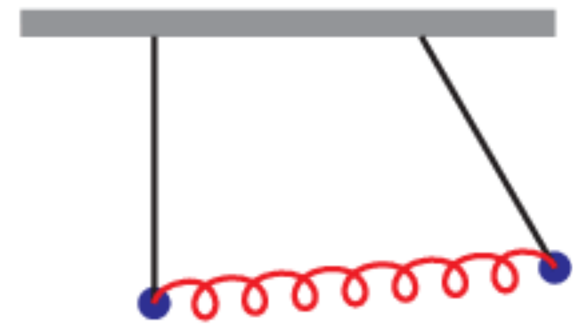

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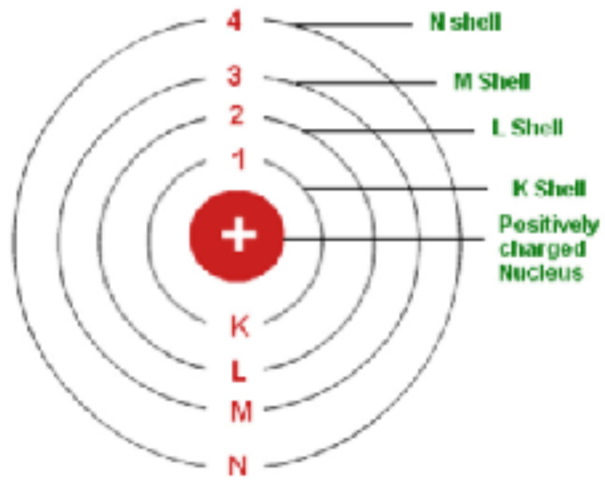

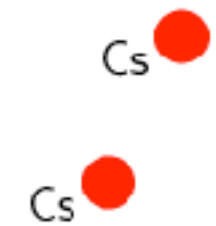

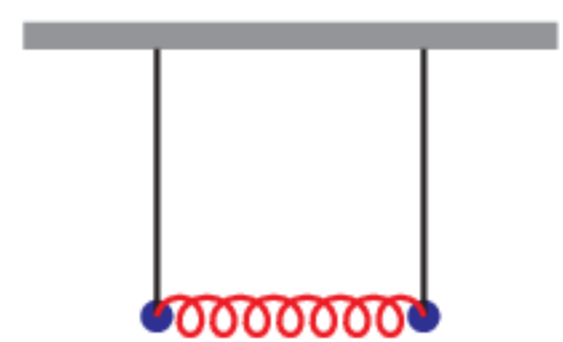

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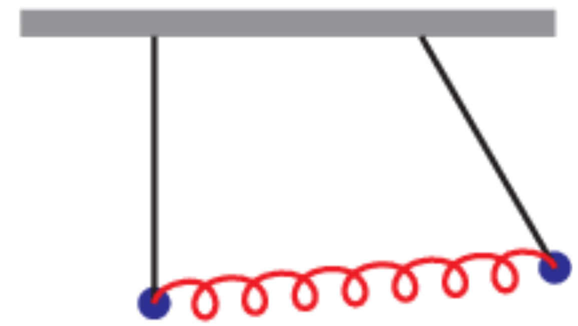

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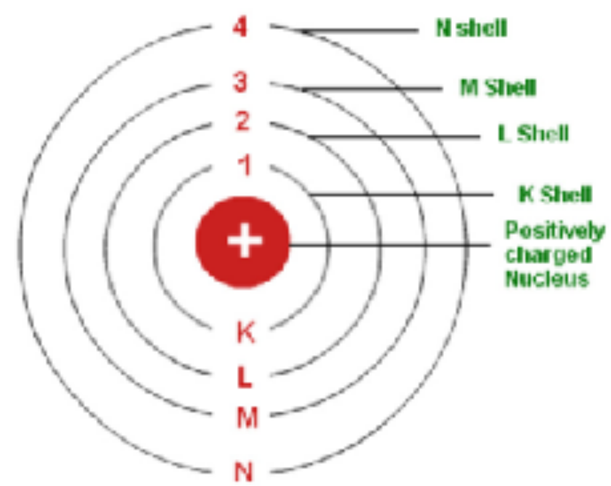
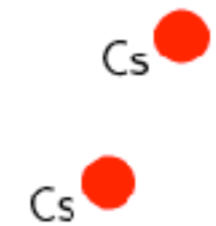
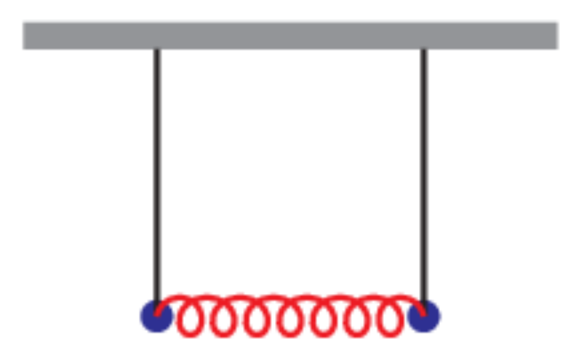




QFT: fields are always and everywhere, but they may not be “excited”



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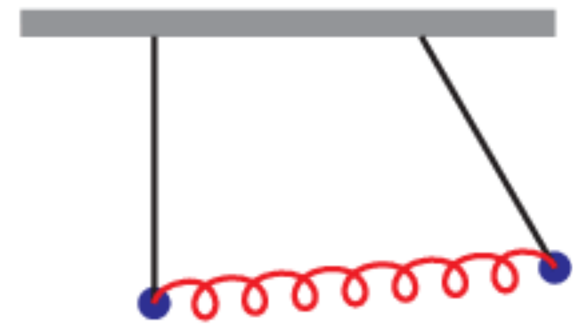

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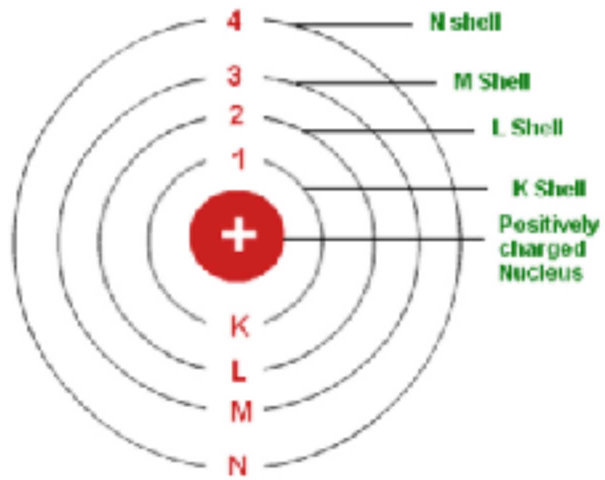

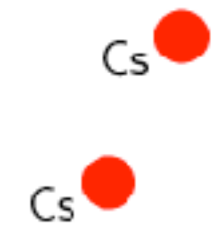

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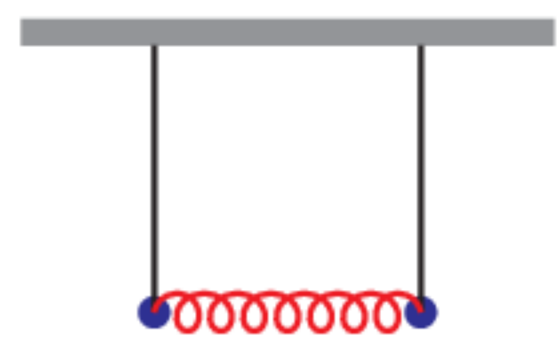

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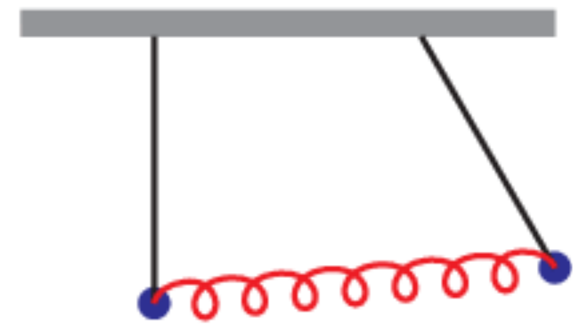

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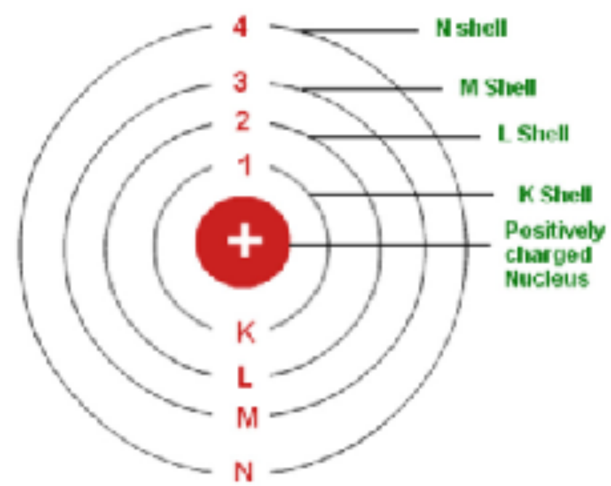

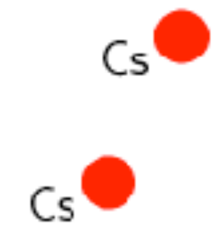

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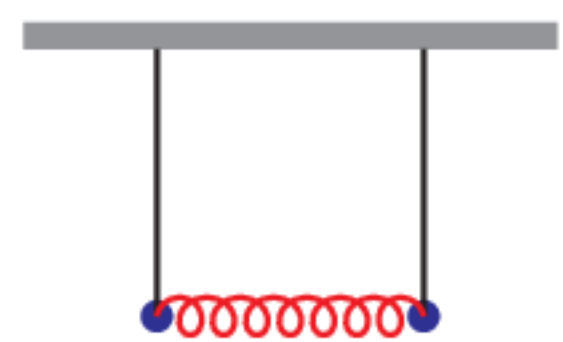
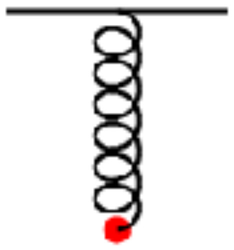


Harmonischer Oszillator



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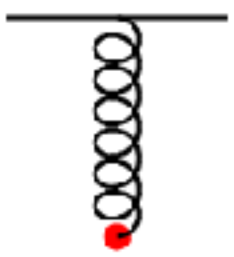


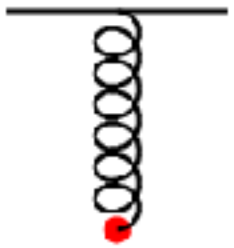
Harmonischer Oszillator

Klassische Mechanik:

$$\ddot{q}(t) + \omega^2 q(t) = 0$$

q : Auslenkung

$\ddot{q}$: Beschleunigung



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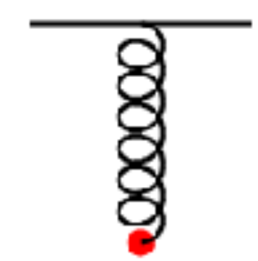

$$q(t) = \tilde{A}\cos(\omega t + \phi)$$

$$E = \frac{1}{2} m \omega^2 \tilde{A}^2$$

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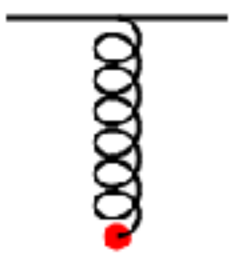

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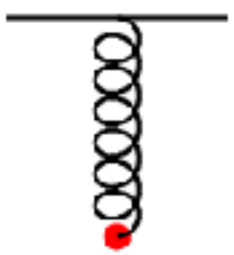

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Quantenmechanik

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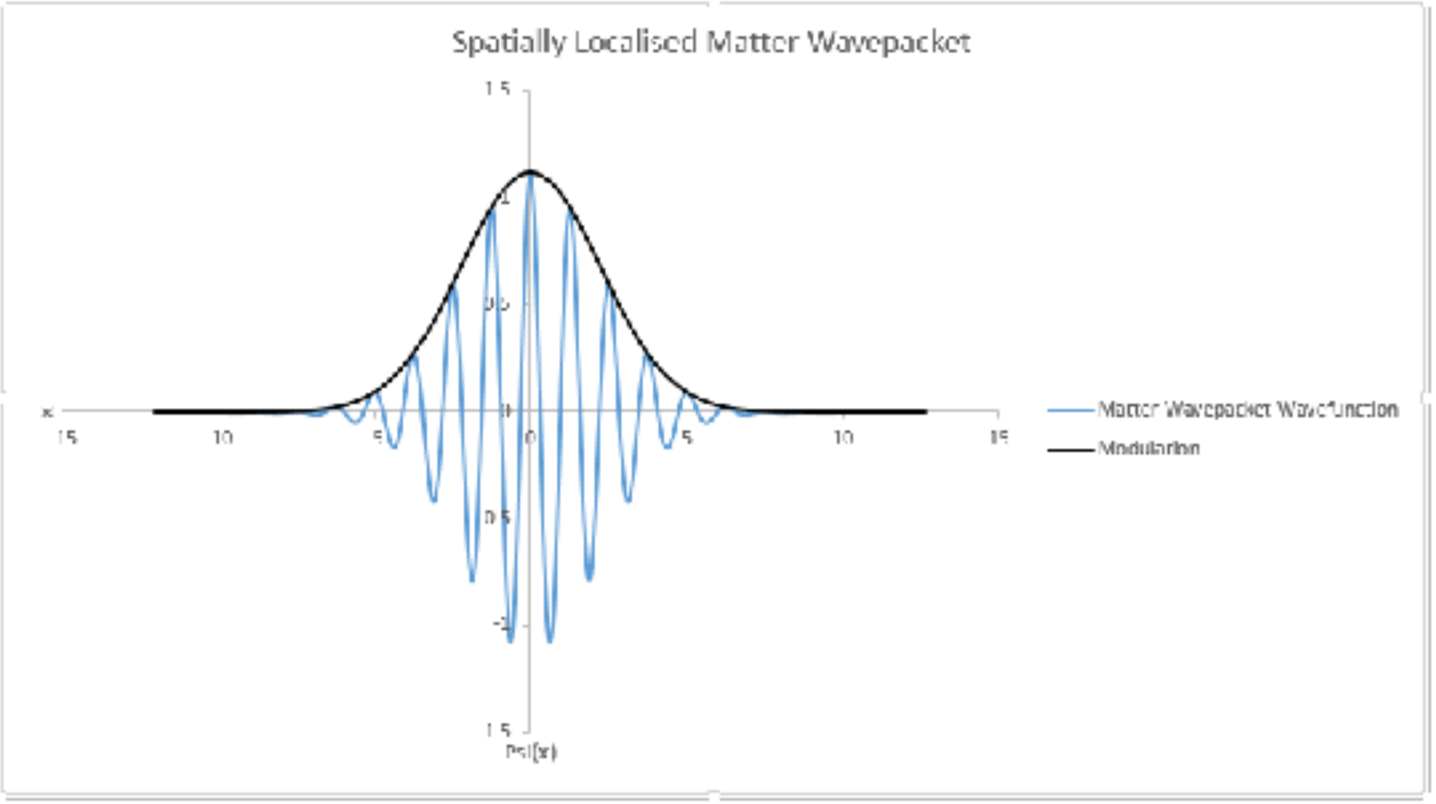
Formalismus der QM

Wellenfunktion: $\psi(x, t)$



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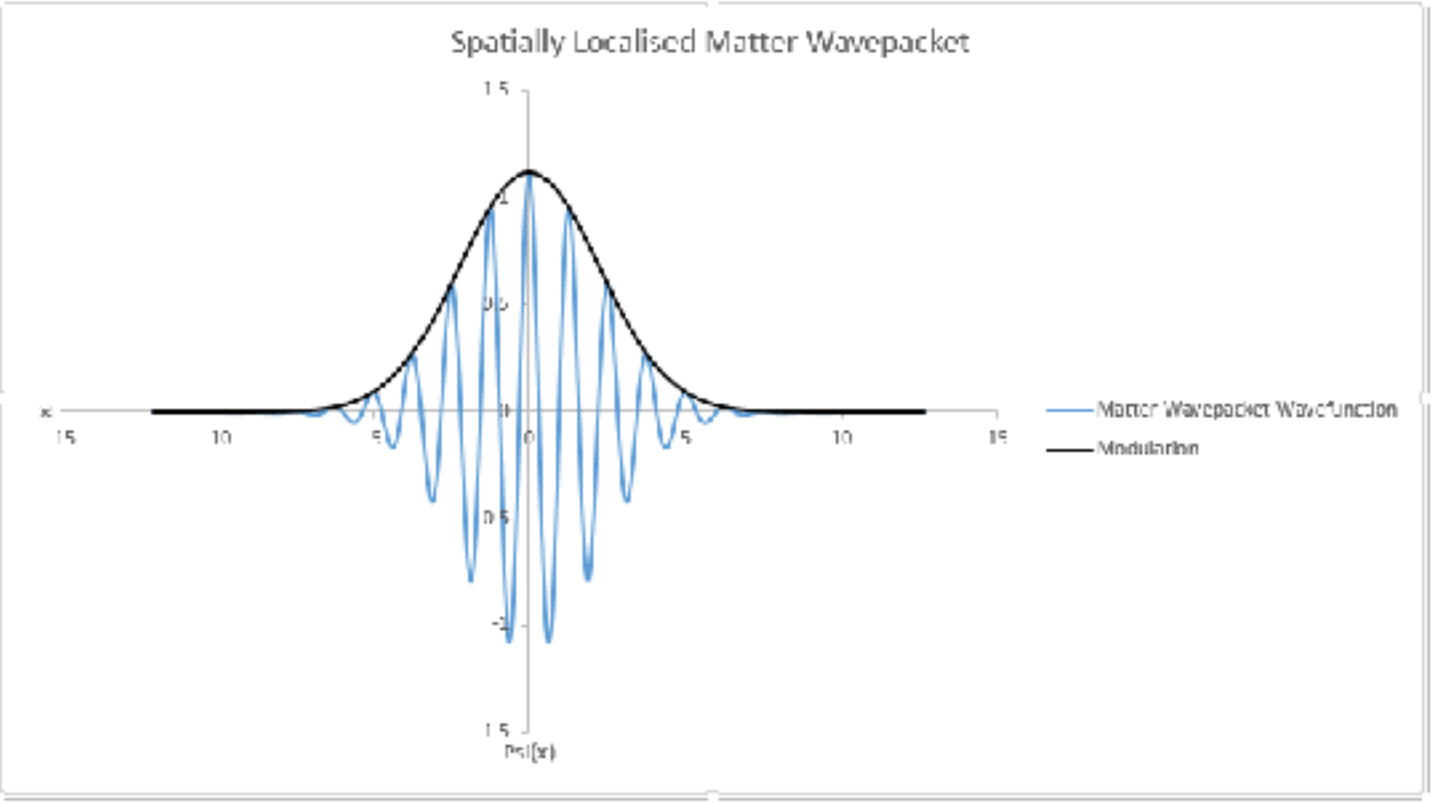
Messgrößen ↔ Operatoren



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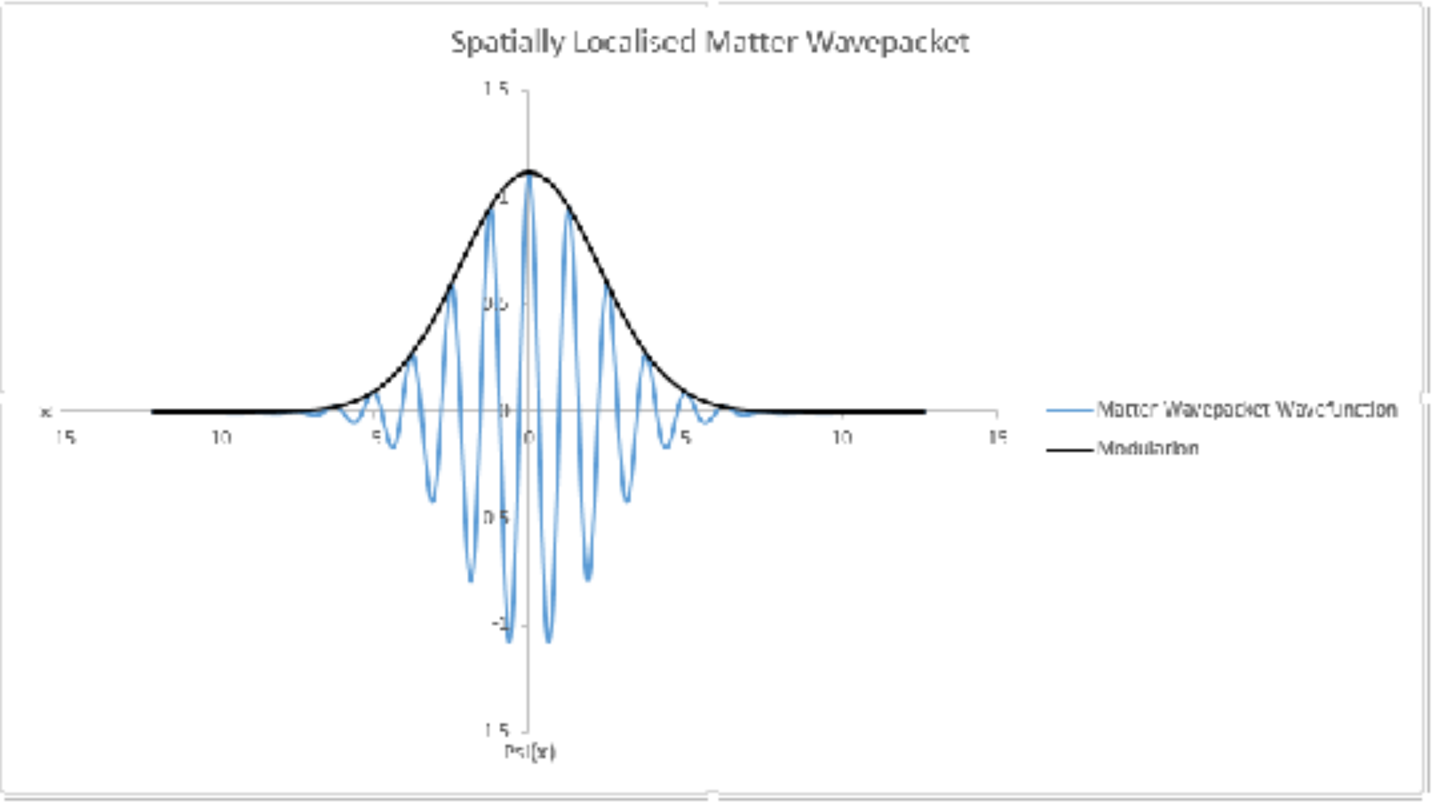

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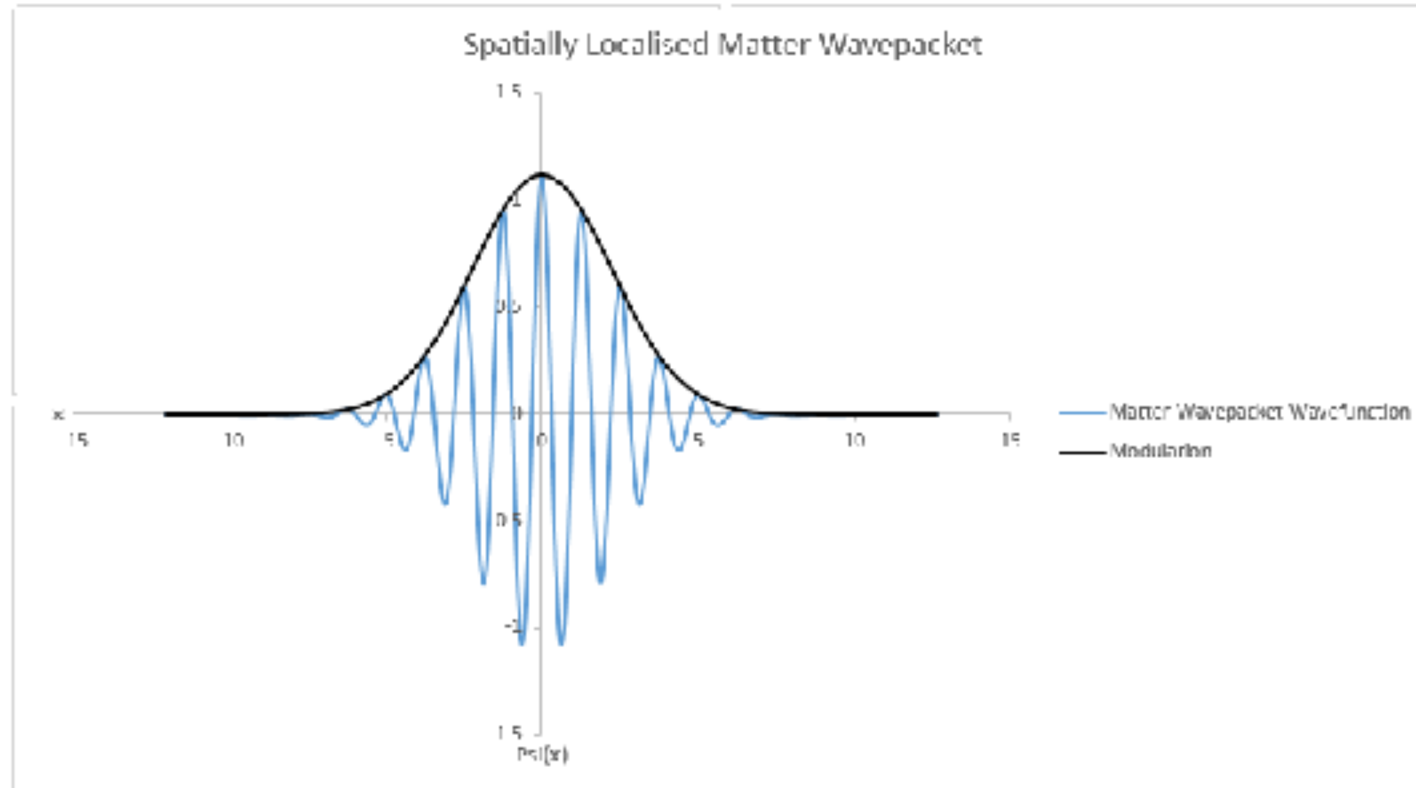

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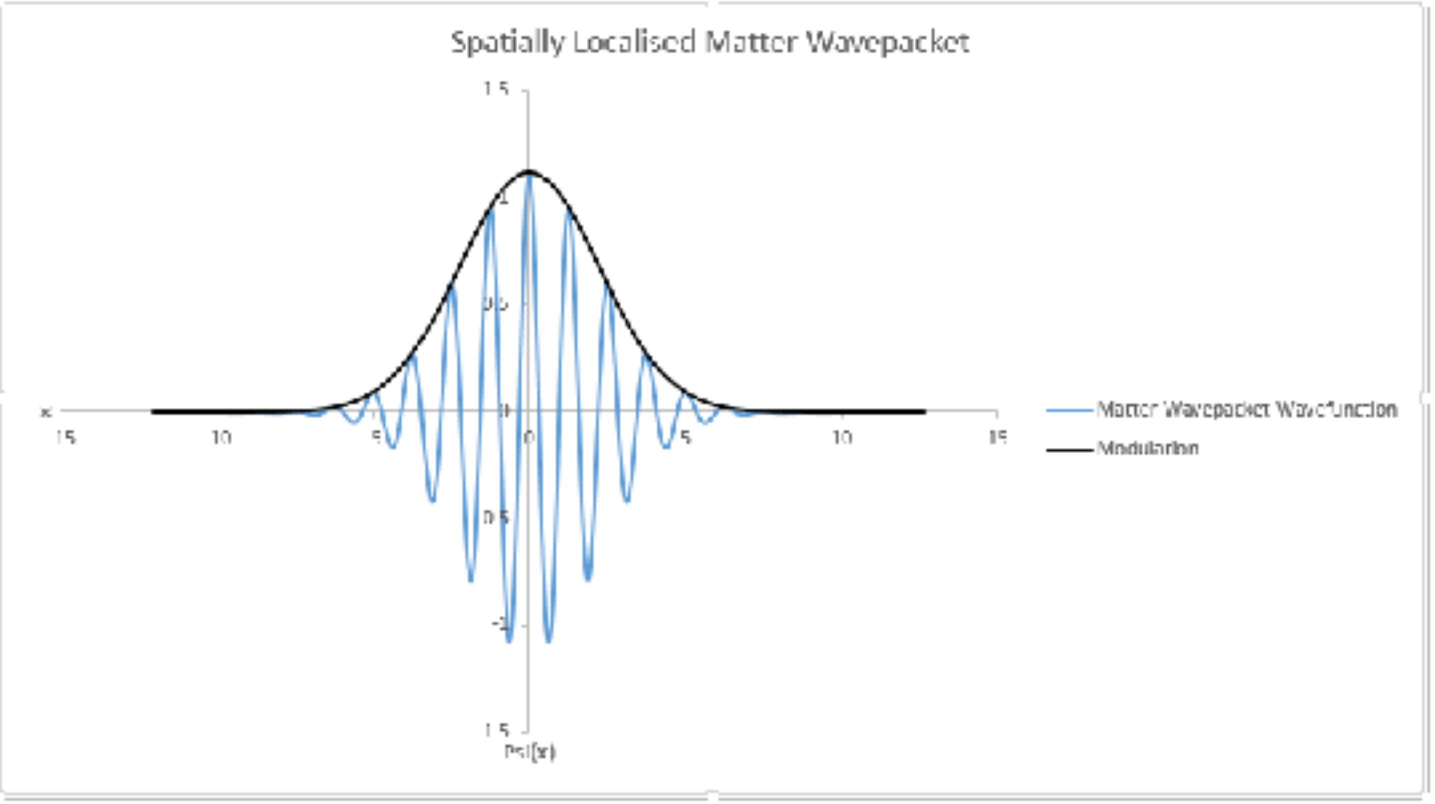

$$\langle\hat{p}(t)\rangle = -i\hbar\int dx\psi^*(x,t)\frac{d}{dx}\psi(x,t)$$

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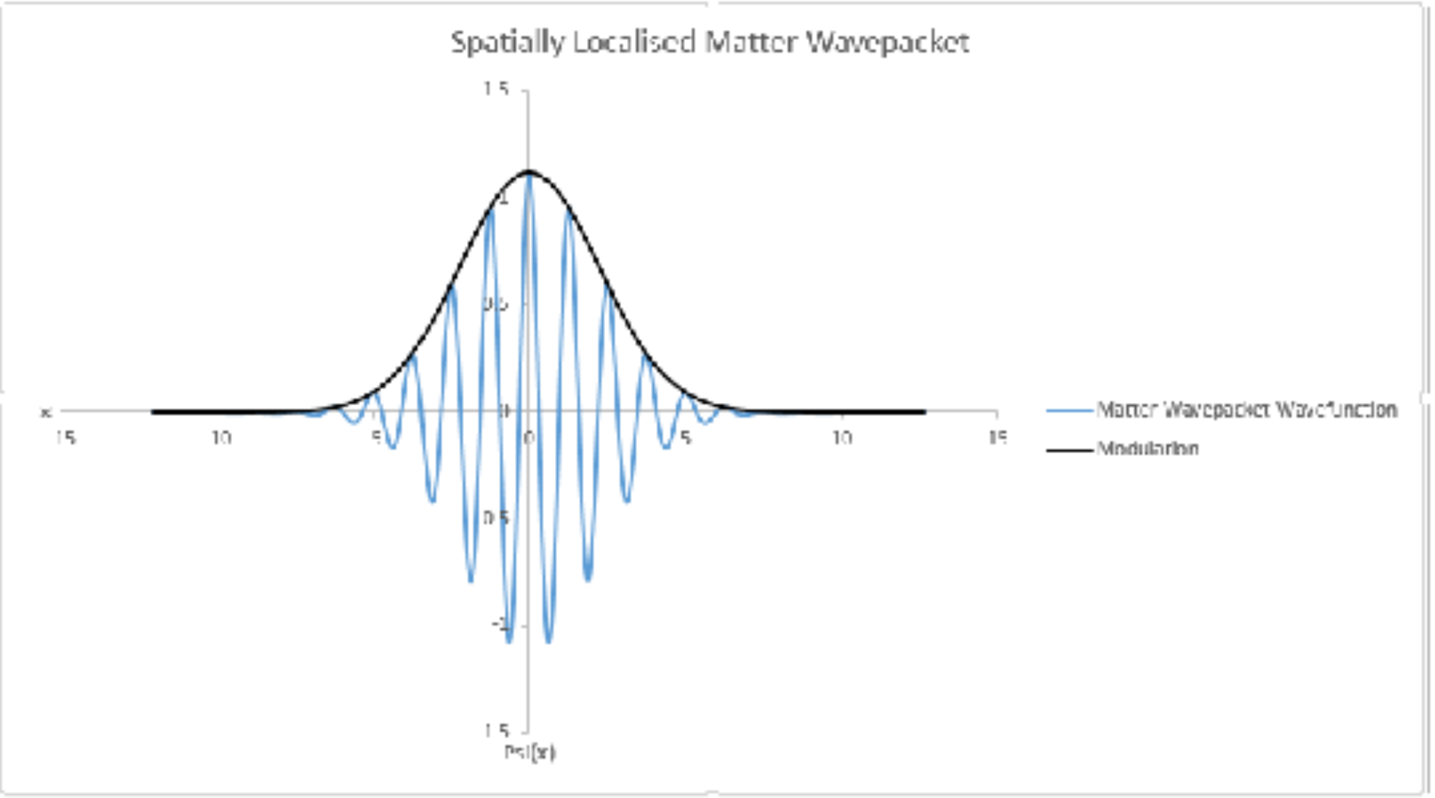

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$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

Ebene Welle

$$\psi_p(x,t) = C_p\, e^{\frac{i}{\hbar}(px - Et)} \qquad E = \frac{p^2}{2m}$$

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unlokalisiertes “Teilchen” mit scharfem Impuls!

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Phasengeschwindigkeit

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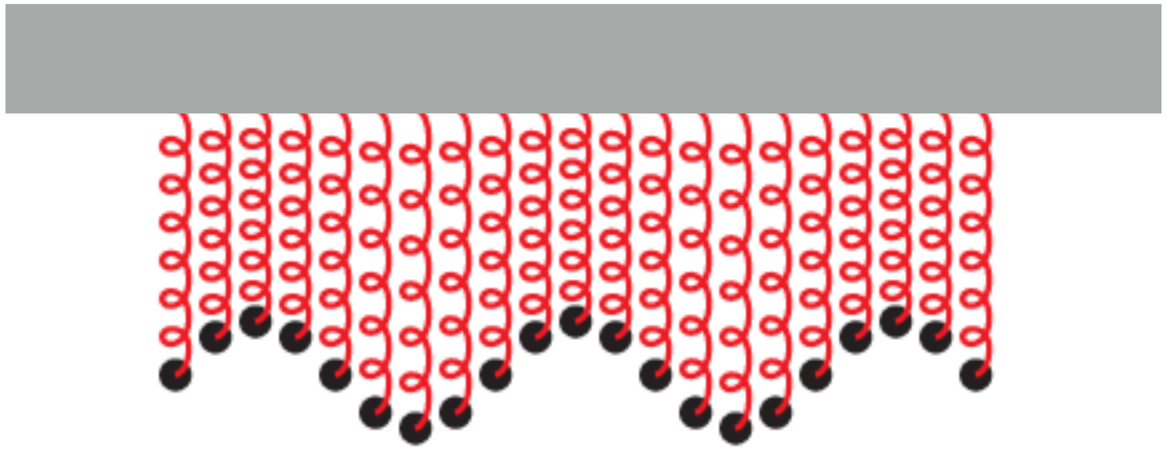

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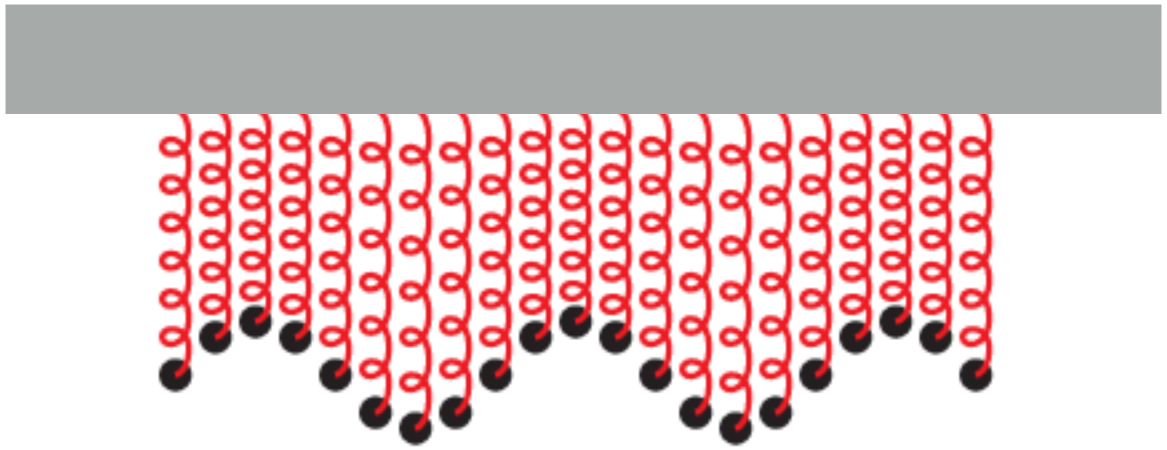

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Quantisiere diese HOs!
"2. Quantisierung"

Nochmal…

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4. Interpretiere dies als klassische HOs an jedem Ort x.

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Quantenfeldtheorie

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Quantenfeldtheorie

(störungstheoretisches Bild)

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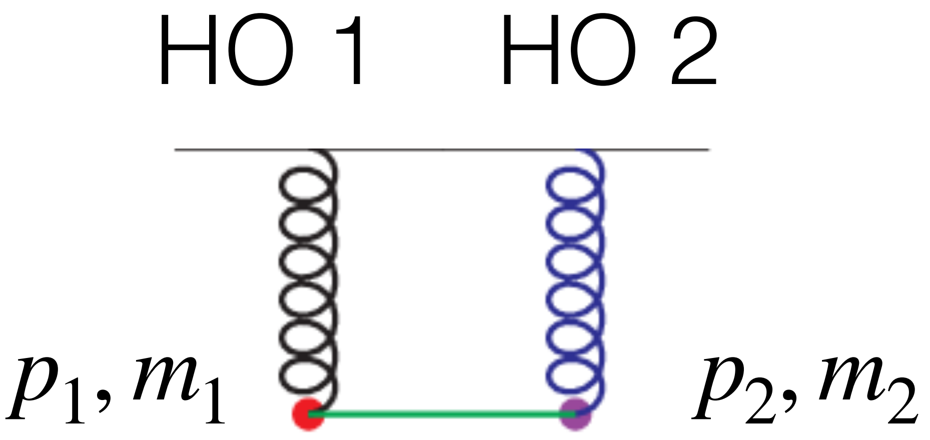
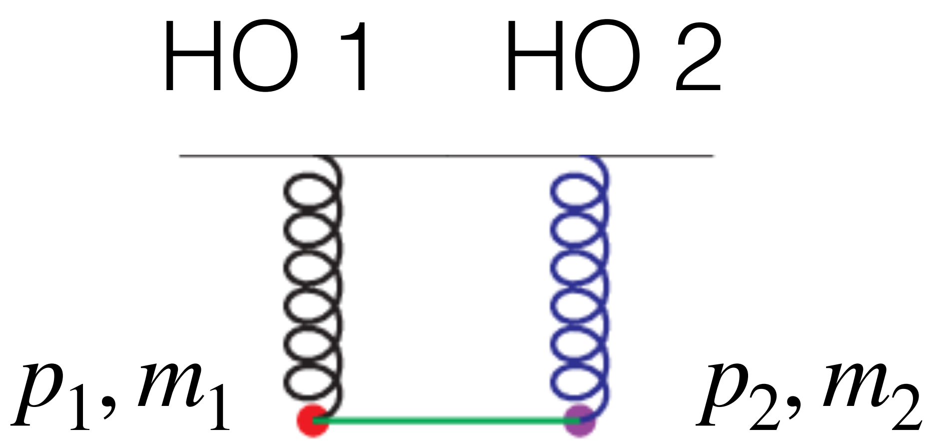
Wechselwirkung



klassisch

Wechselwirkung



klassisch

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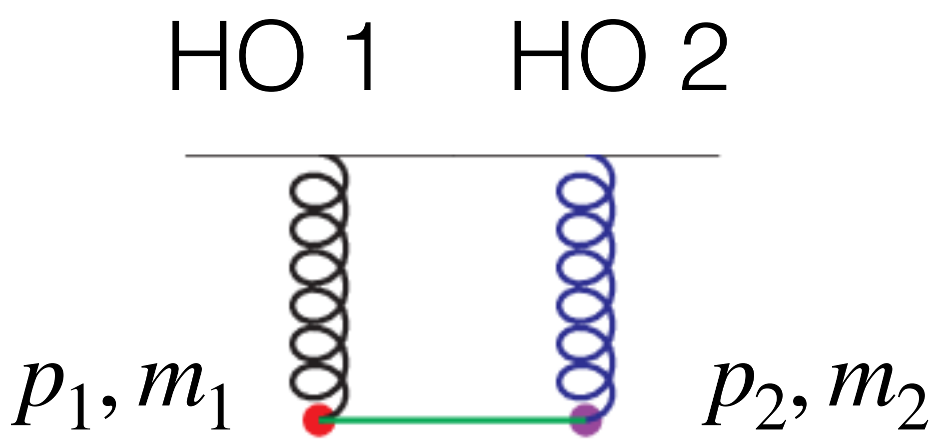
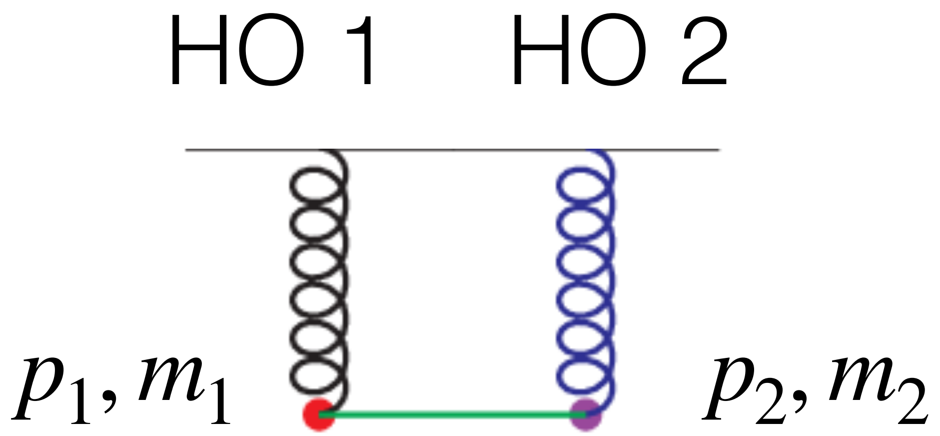
Wechselwirkung



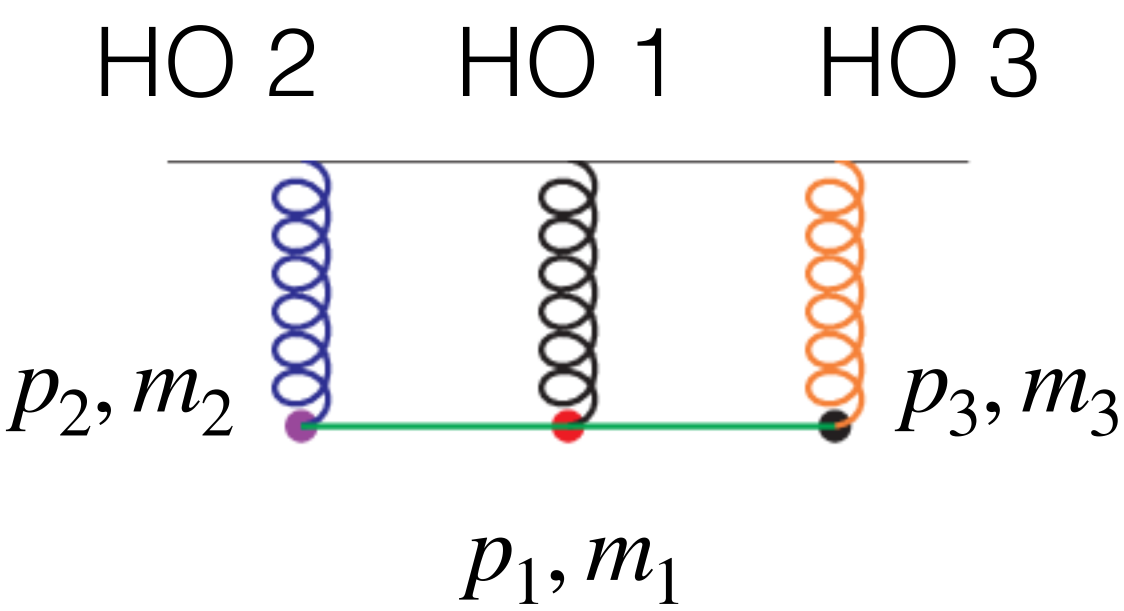
quantenmechanisch

Wechselwirkung



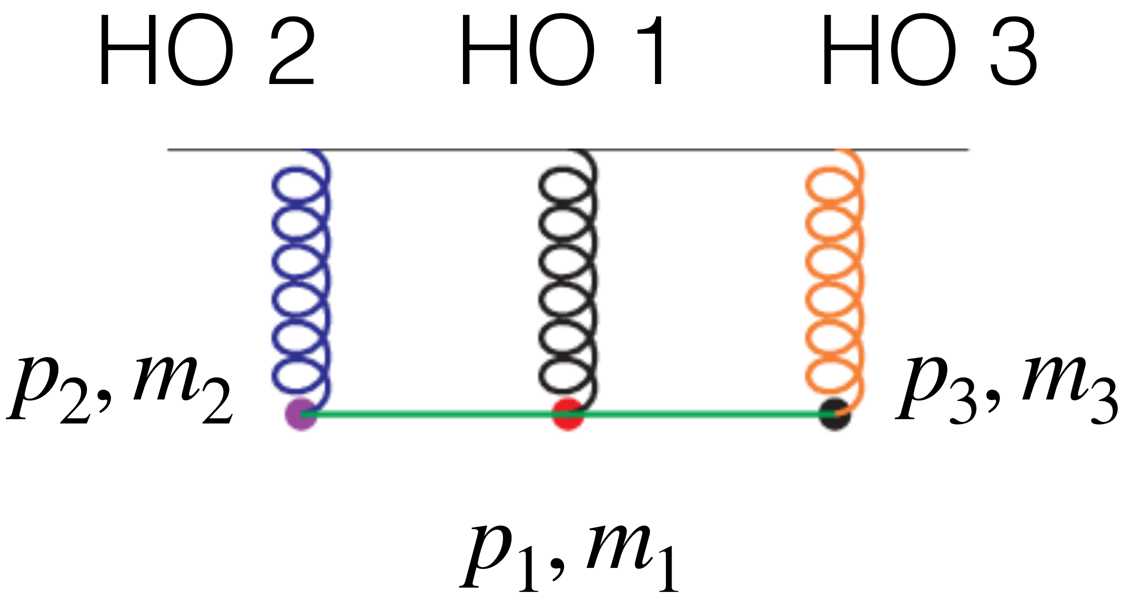
quantenmechanisch

Wechselwirkung



quantenmechanisch

Wechselwirkung



quantenmechanisch

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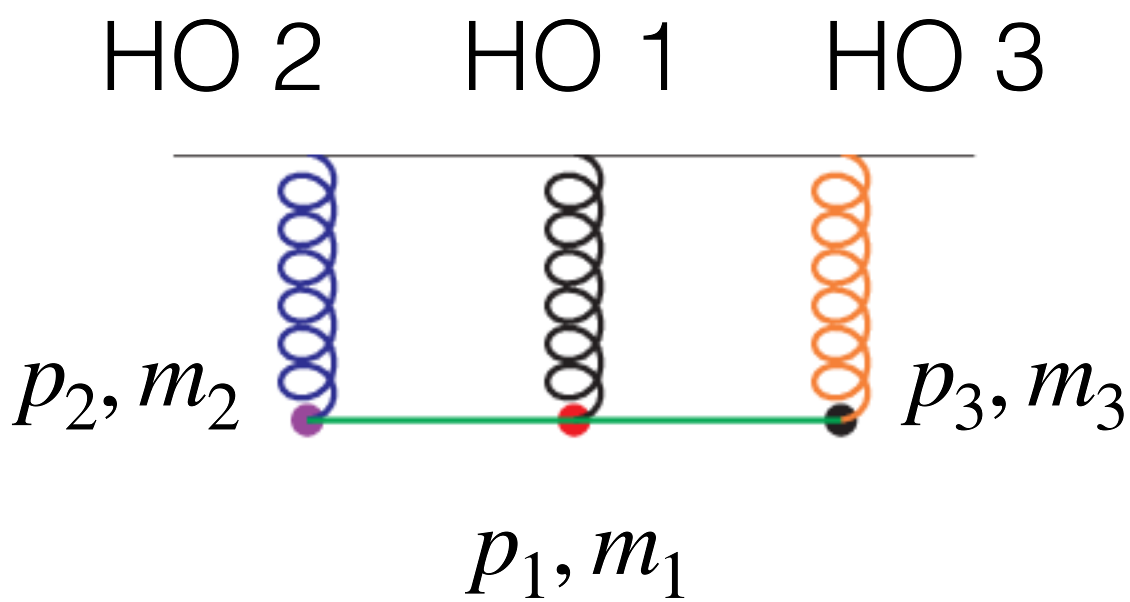
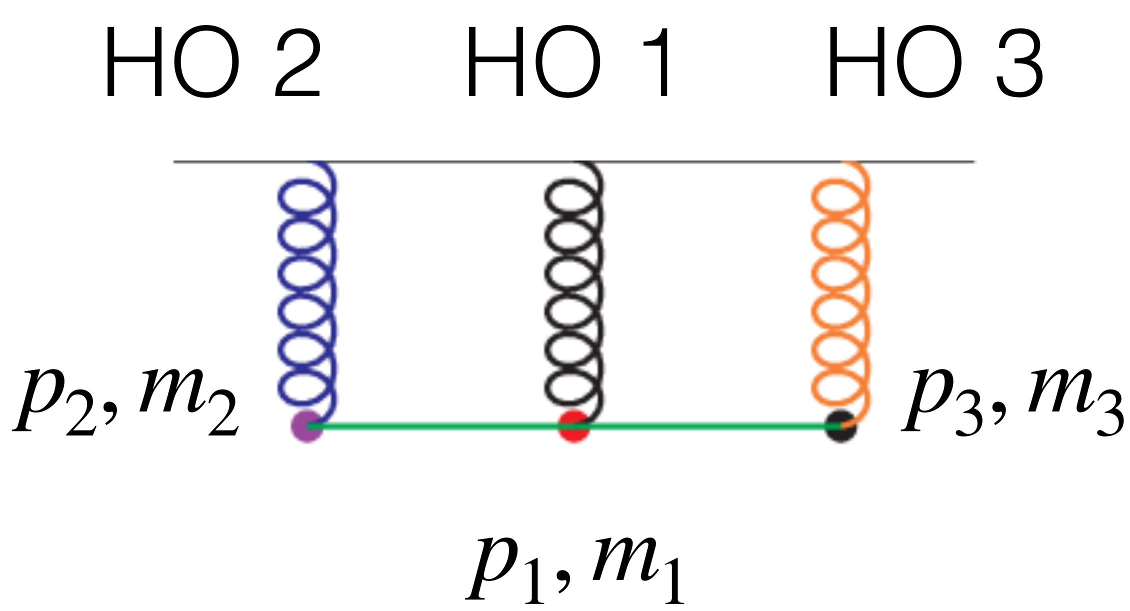
Wechselwirkung



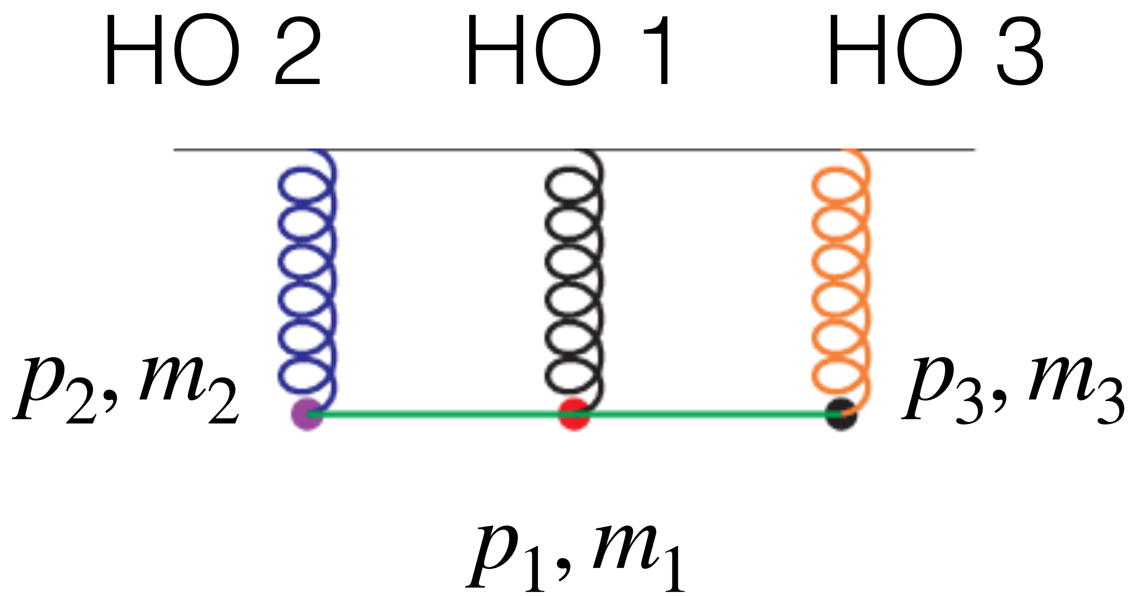
quantenmechanisch

Wechselwirkung



quantenmechanisch

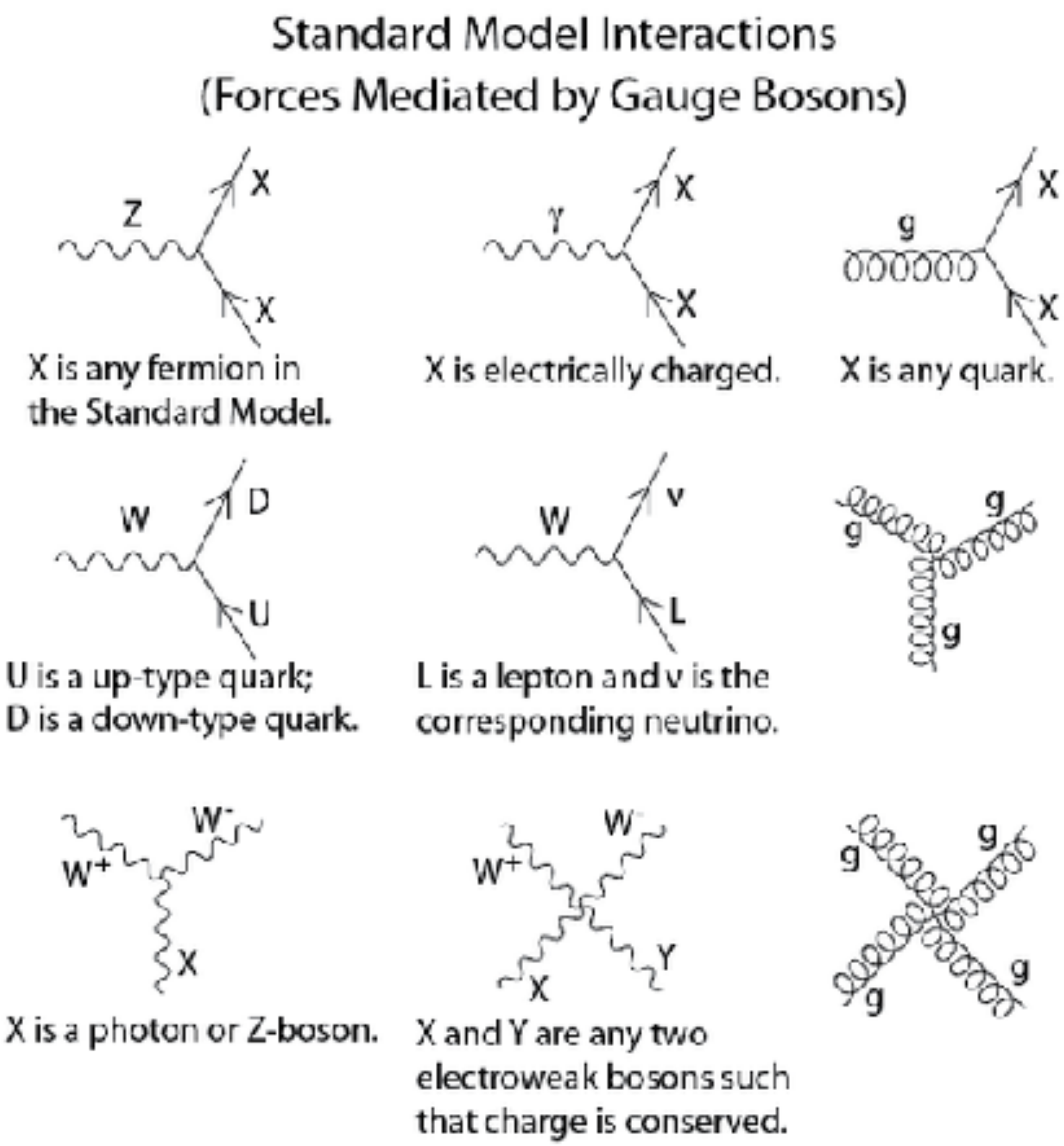
Wechselwirkung

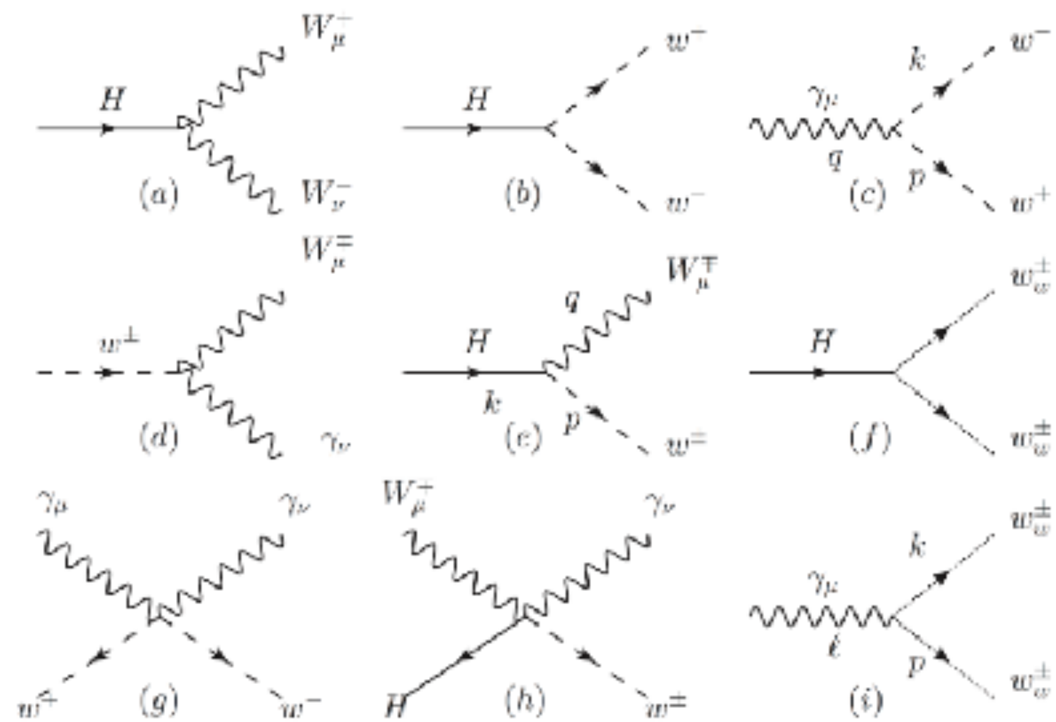


quantenmechanisch

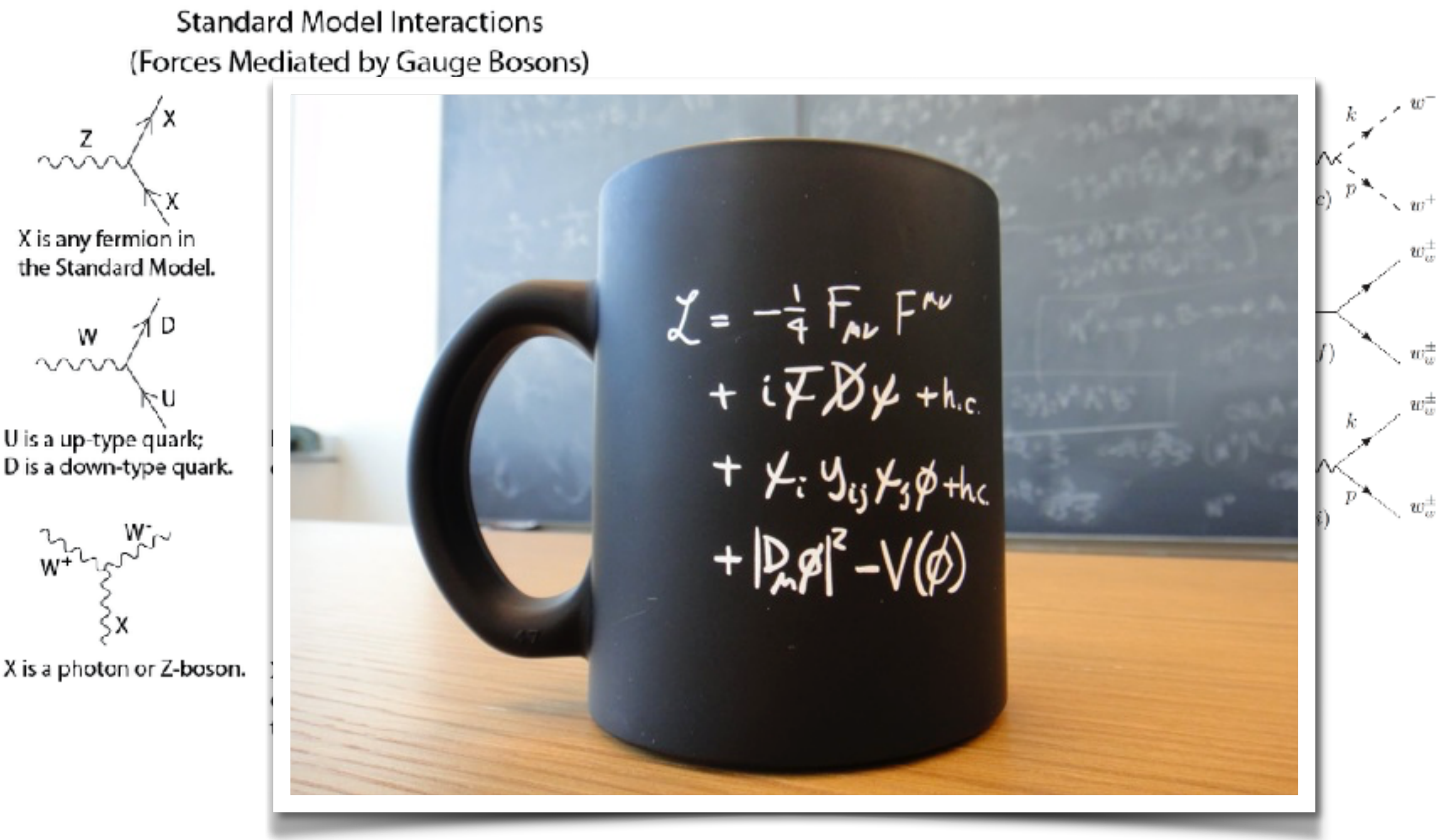
Vertex

Vertices

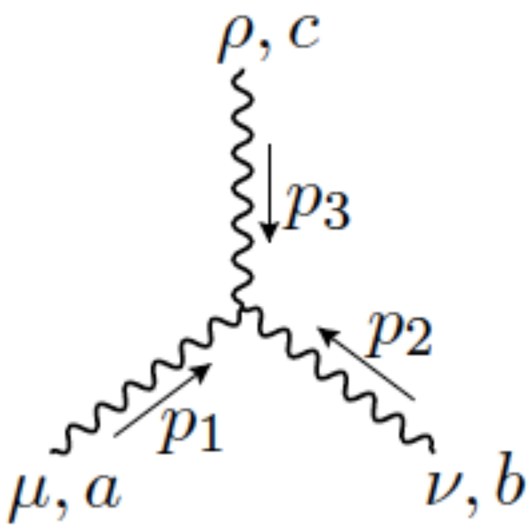

Standard Model Interactions
(Forces Mediated by Gauge Bosons)
Z
X
X
X is any fermion in the Standard Model.
γ
X
X
X is electrically charged.
g
X
X
X is any quark.
W
D
U
U is a up-type quark; D is a down-type quark.
W
v
L
L is a lepton and v is the corresponding neutrino.
g
g
g
W+
W-
X
X is a photon or Z-boson.
W+
W-
X
Y
X and Y are any two electroweak bosons such that charge is conserved.
g
g
g
g


H
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)

Vertices

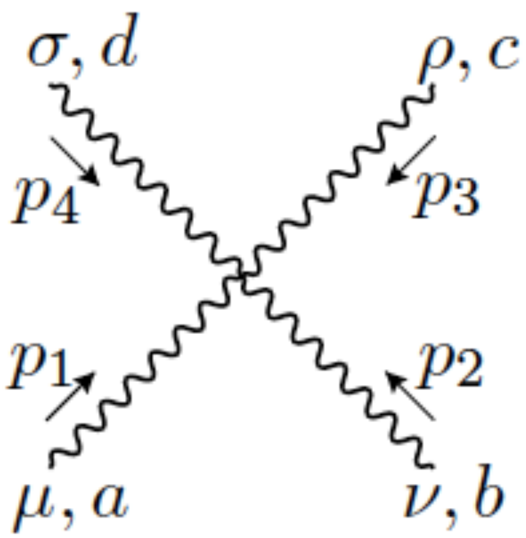

Standard Model Interactions
(Forces Mediated by Gauge Bosons)
Z
X
X
X is any fermion in the Standard Model.
W
D
U
U is a up-type quark;
D is a down-type quark.
W+
W-
X
X is a photon or Z-boson.
$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$+ i \bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$

Feynman-Regeln

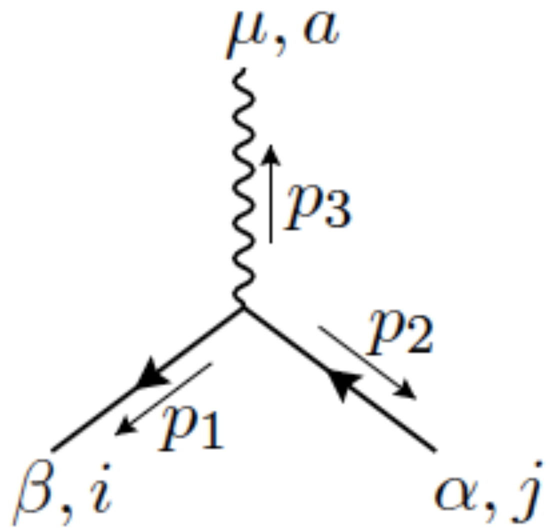


$$g f^{abc}\left[\; g^{\mu\nu}(p_1 - p_2)^\rho + g^{\nu\rho}(p_2 - p_3)^\mu + g^{\rho\mu}(p_3 - p_1)^\nu \right]$$

Feynman-Regeln



$$-ig^2\Big[\; f_{eab}f_{ecd}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) + f_{eac}f_{edb}(g_{\mu\sigma}g_{\rho\nu} - g_{\mu\nu}g_{\rho\sigma}) + f_{ead}f_{ebc}(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma})\Big]$$



$$ig(\gamma^\mu)_{\beta\alpha}T^a_{ij}$$

Myon-Wechselwirkungen

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Myon-Wechselwirkungen

Myon-Wechselwirkungen

W-Boson

Myon

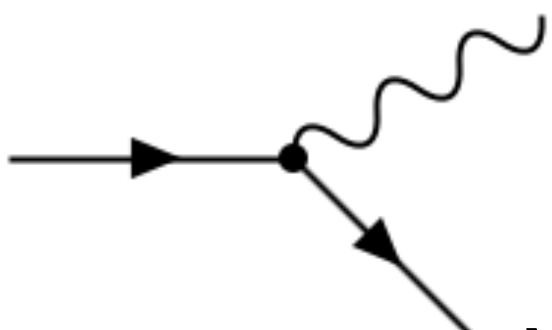
Myon-Neutrino

Myon-Wechselwirkungen

W-Boson

Myon •

Myon-Neutrino

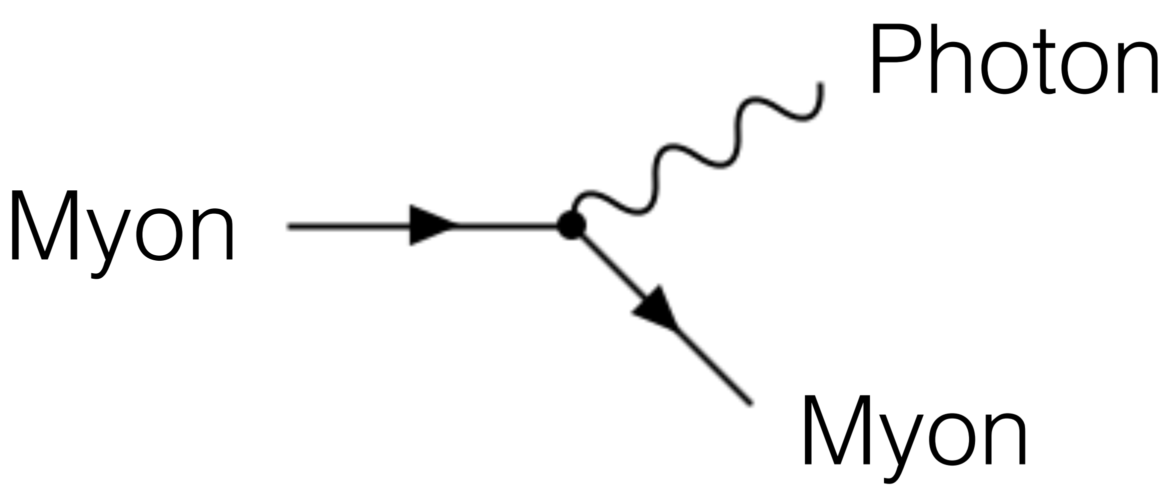


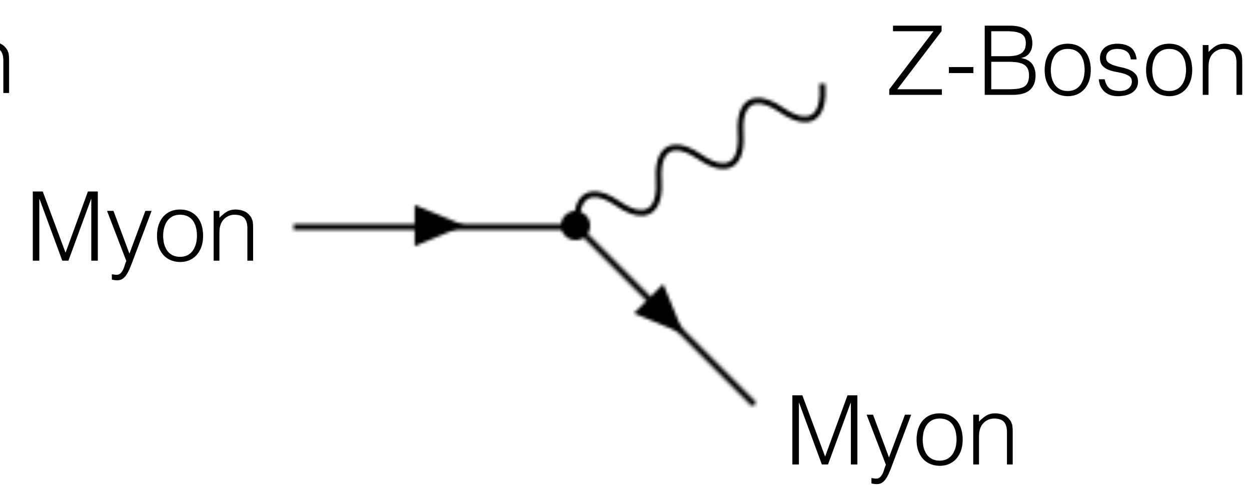
Myon-Wechselwirkungen

W-Boson

Myon •

Myon-Neutrino





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Myon-Zerfall?

W-Boson

Myon •

Myon-Neutrino

Myon-Zerfall?

W-Boson

Myon

Myon-Neutrino

Myon-Zerfall?

80.000 MeV/c²

W-Boson

Myon •

100 MeV/c²

Myon-Neutrino

0 MeV/c²

Myon-Zerfall?

80.000 MeV/c²

W-Boson

Myon •

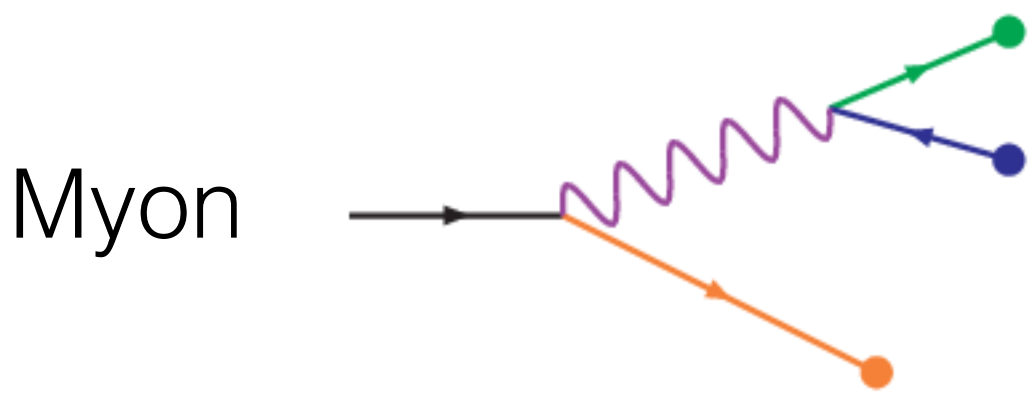
100 MeV/c²

Myon-Neutrino

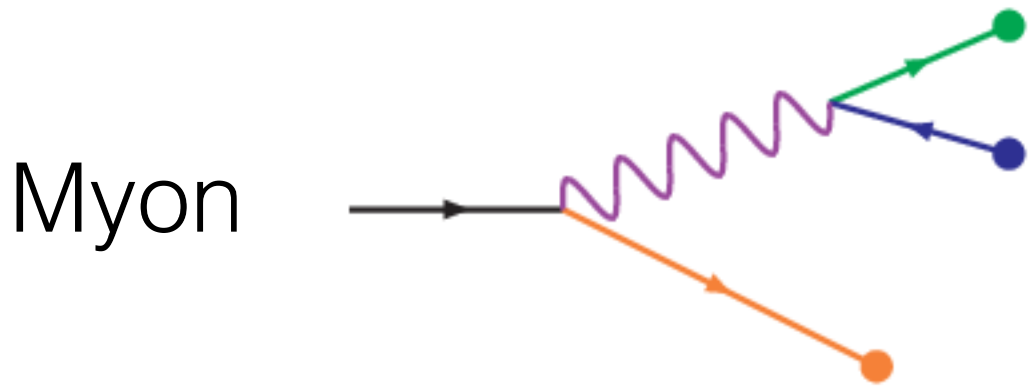
0 MeV/c²

QM: $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$ $\qquad$ $\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$

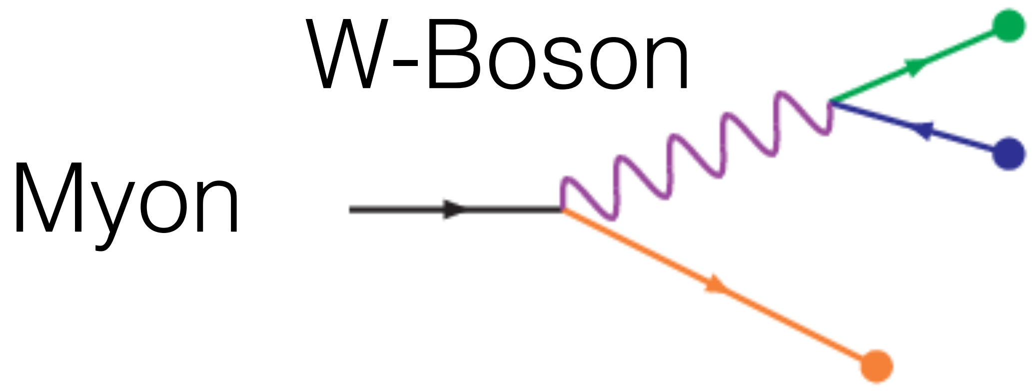
Myon-Zerfall?



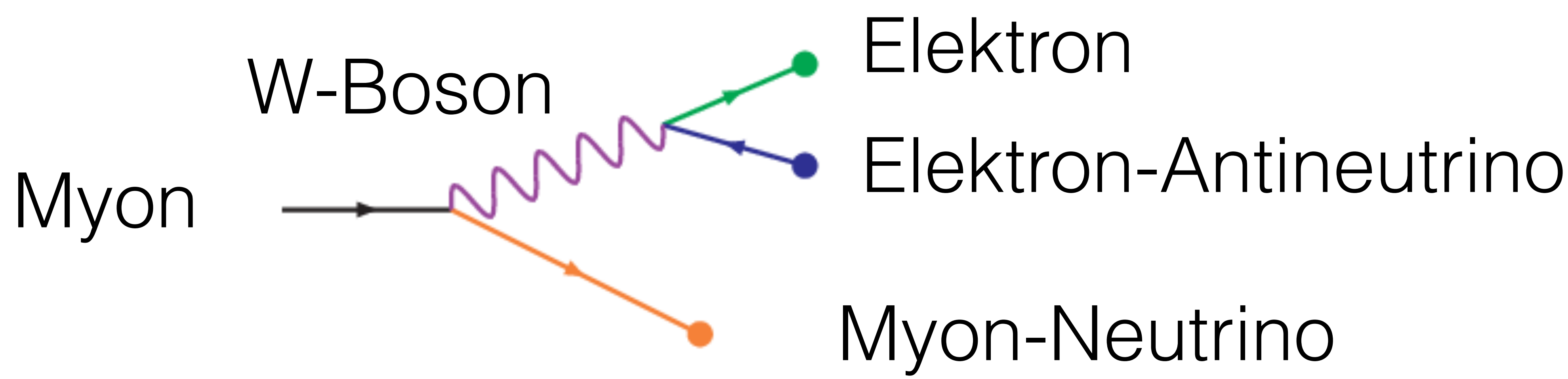
Myon-Zerfall?



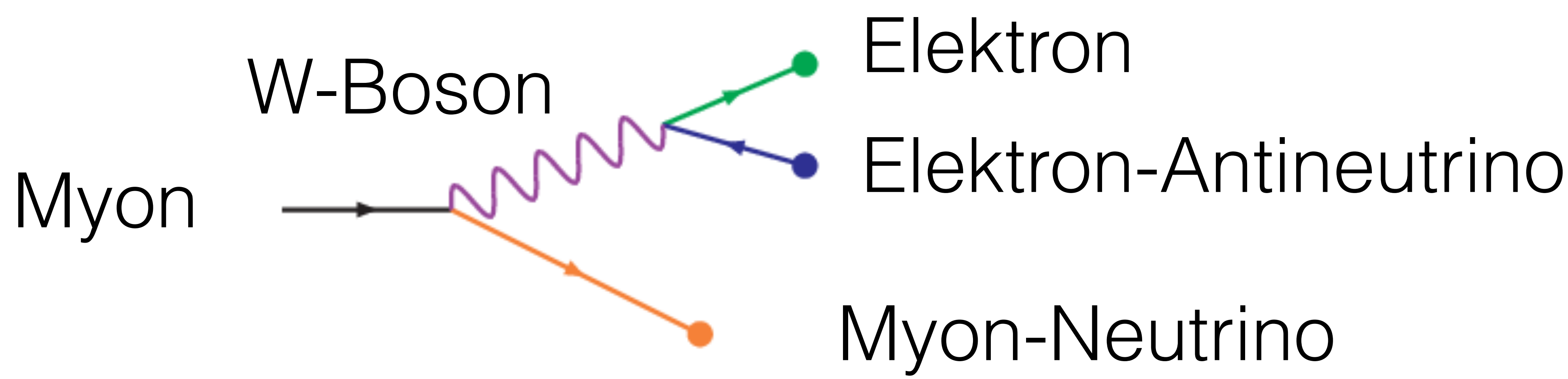
Myon-Zerfall?



Myon-Zerfall?



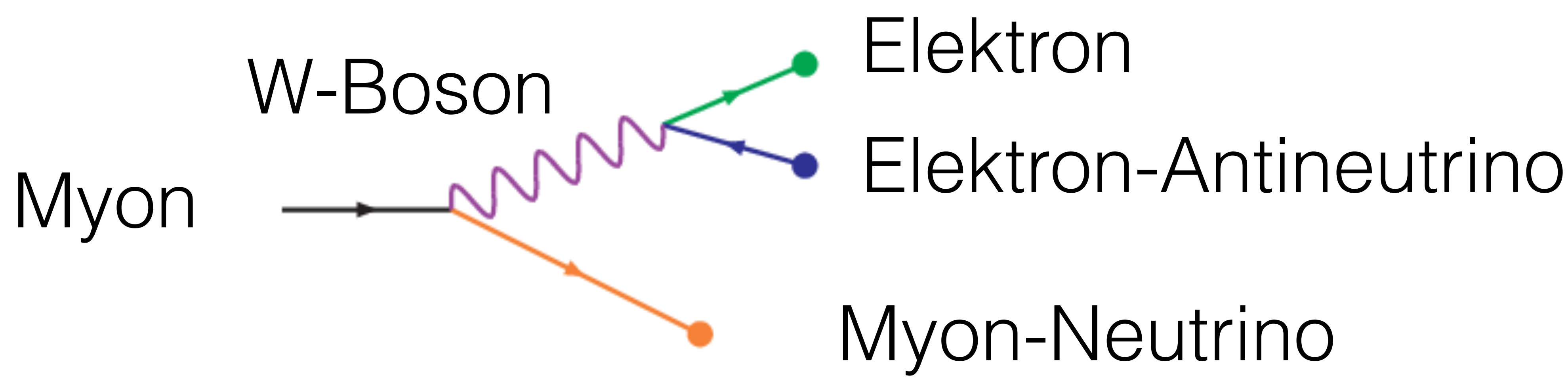
Myon-Zerfall?



W-Boson ist “off-shell”:

$$\Delta E^2 \equiv E^2 - E_W^2 = E^2 - M_W^2 c^4 - p_W^2 c^2 \neq 0$$

Myon-Zerfall?

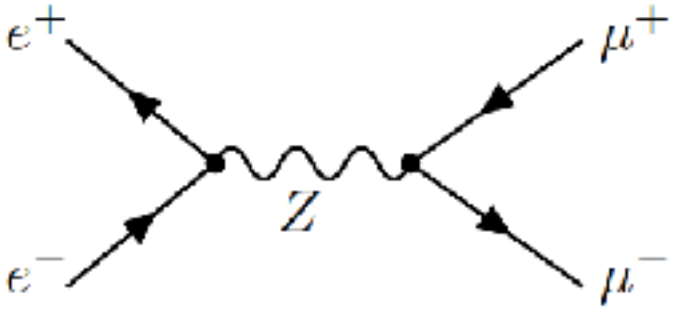


W-Boson ist “off-shell”:

$$\Delta E^2 \equiv E^2 - E_W^2 = E^2 - M_W^2 c^4 - p_W^2 c^2 \neq 0$$

“Off-shellness” unterdrückt mit $1/(\Delta E^2)$

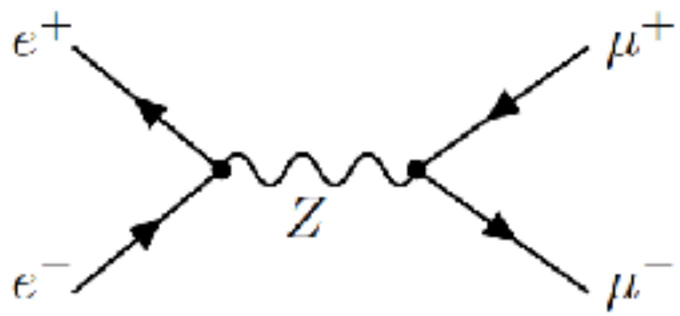
Wirkungsquerschnitte

$E \rightarrow$ 

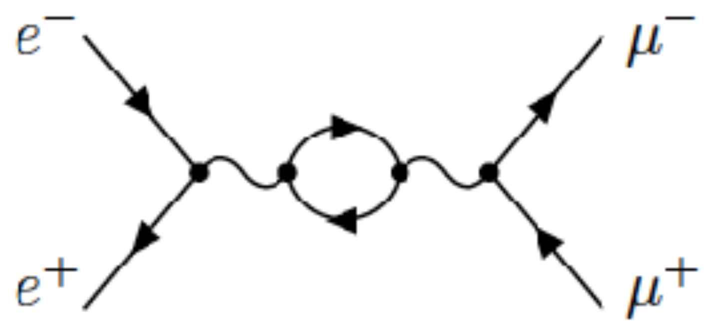
$$\sigma \sim \frac{1}{|E^2 - M_Z^2 c^4|^2}$$

Wirkungsquerschnitte

$E \rightarrow$



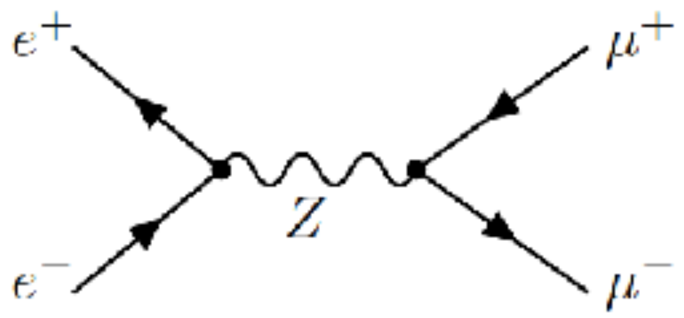
$$\sigma \sim \frac{1}{|E^2 - M_Z^2 c^4|^2}$$



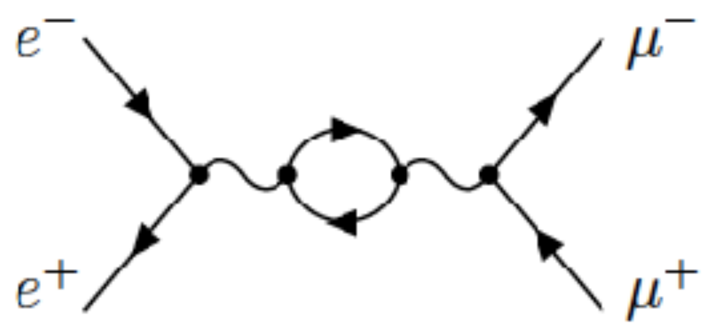
$$\sigma \sim \frac{1}{(E^2 - M_Z^2 c^4)^2 + M_Z^2 \Gamma_Z^2}$$

Wirkungsquerschnitte

$E \rightarrow$



$$\sigma \sim \frac{1}{|E^2 - M_Z^2 c^4|^2}$$



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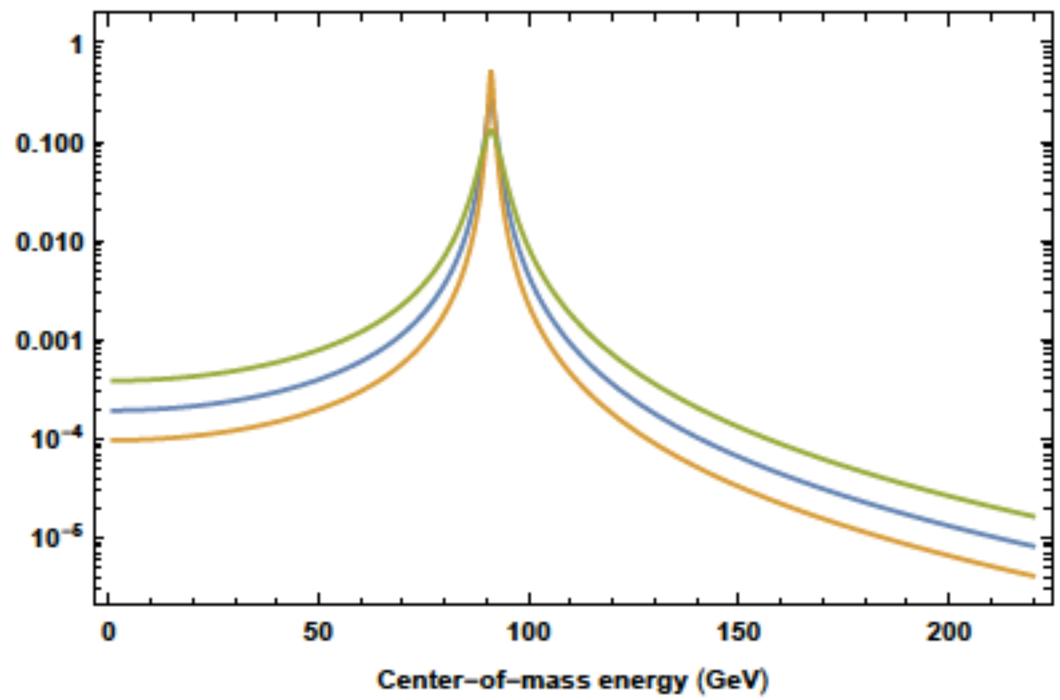
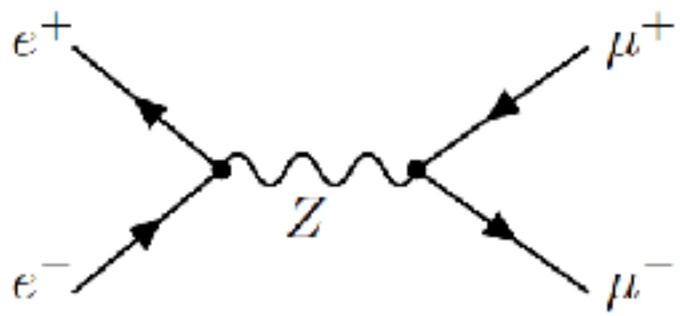


Wirkungsquerschnitte

$E \rightarrow$



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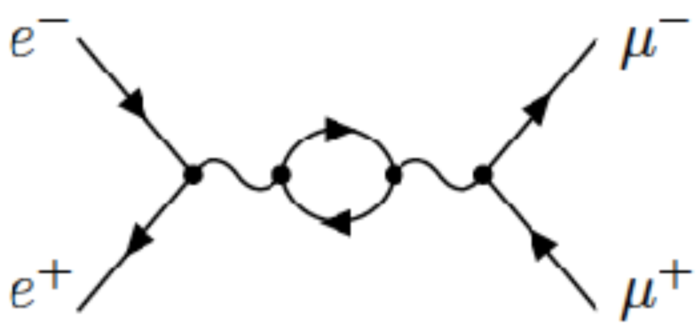

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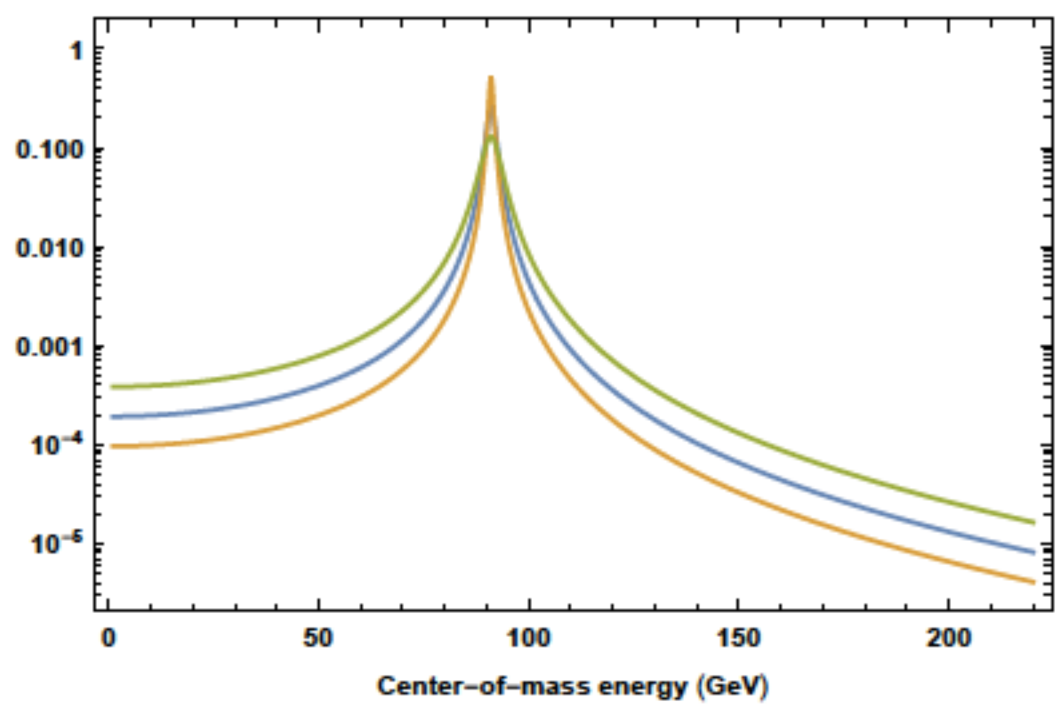
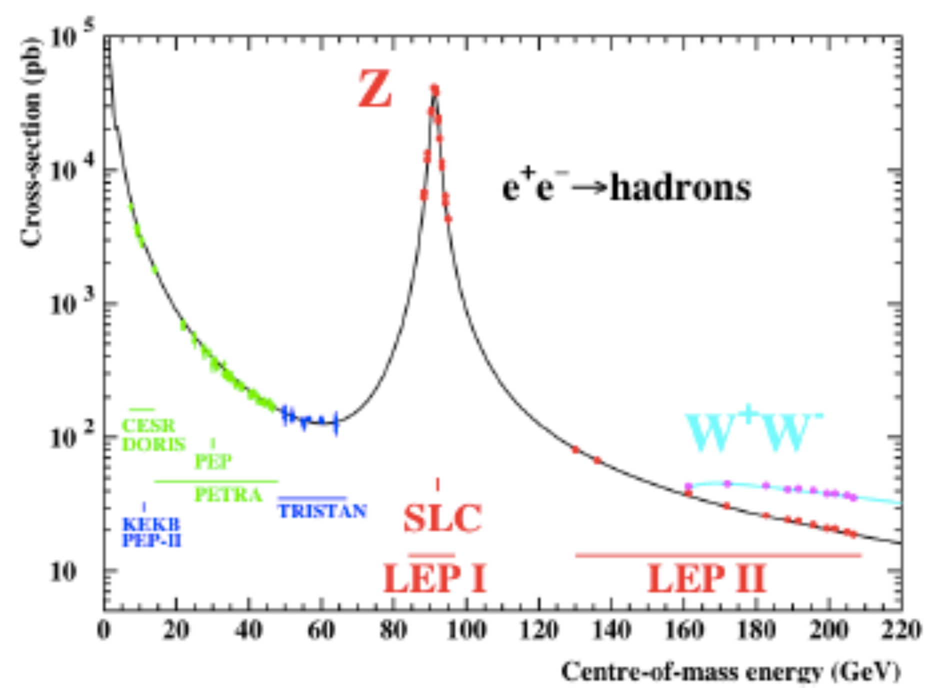
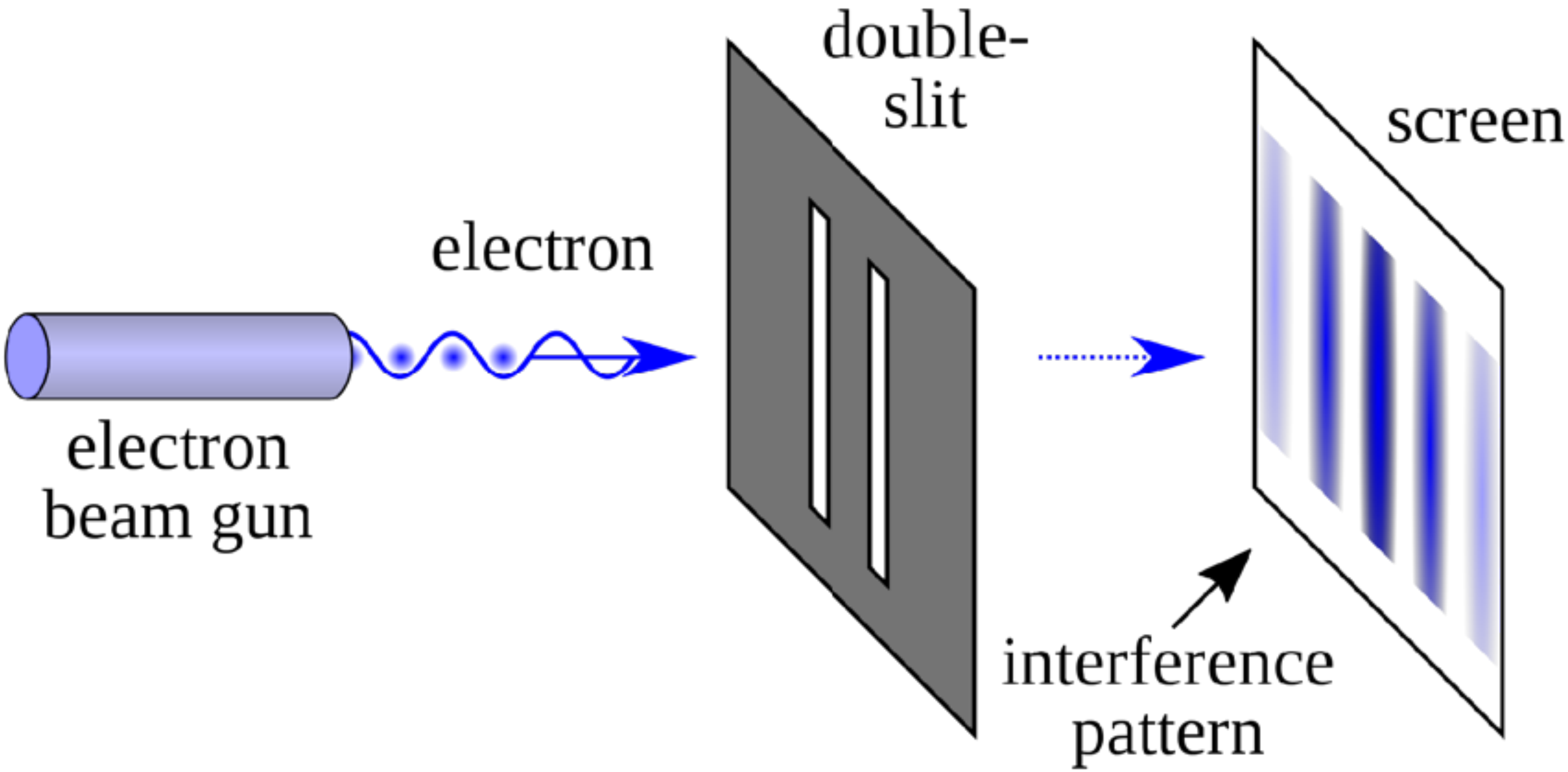


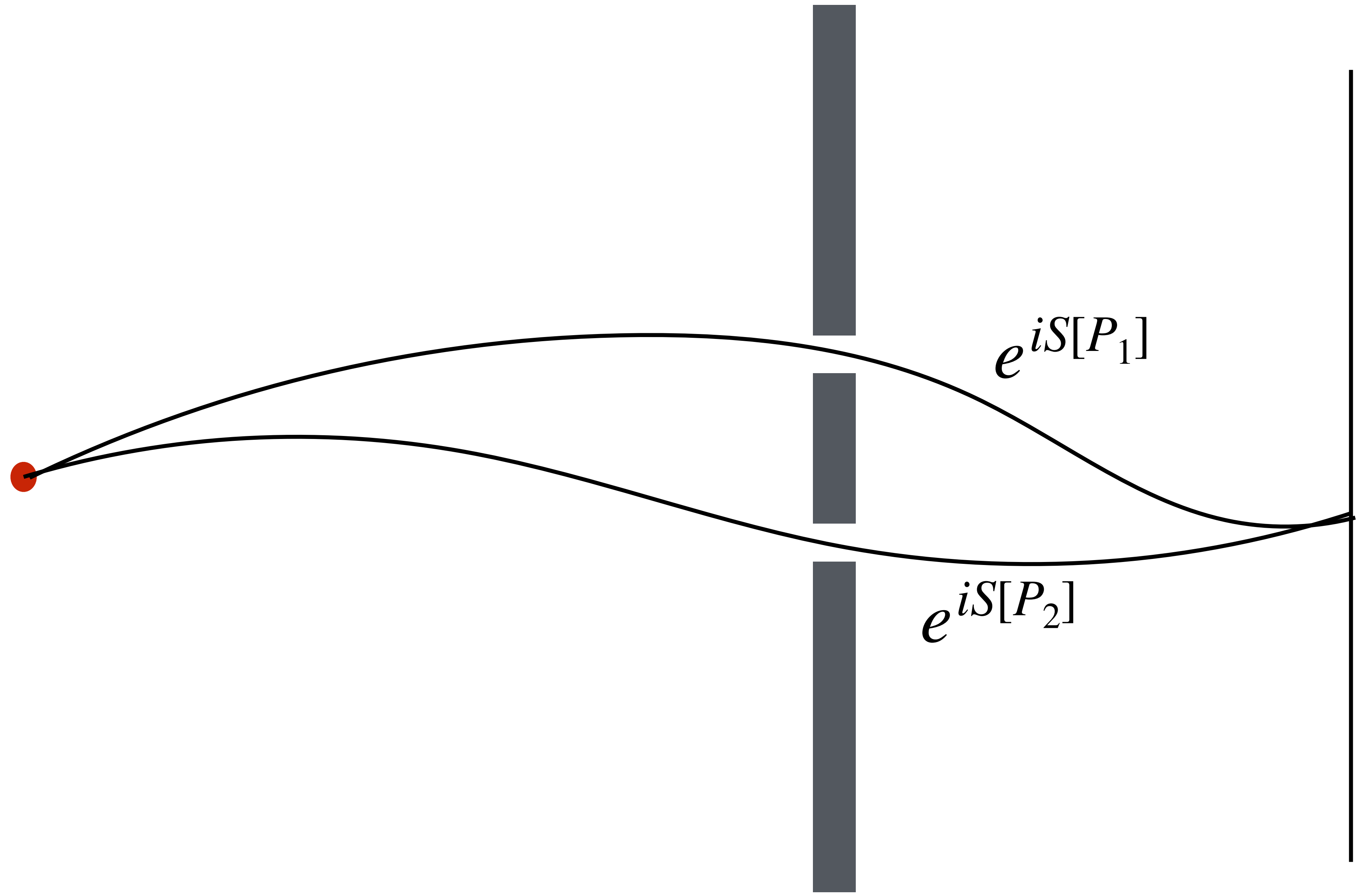


Intermezzo



electron
double-
slit
screen
electron
beam gun
interference
pattern

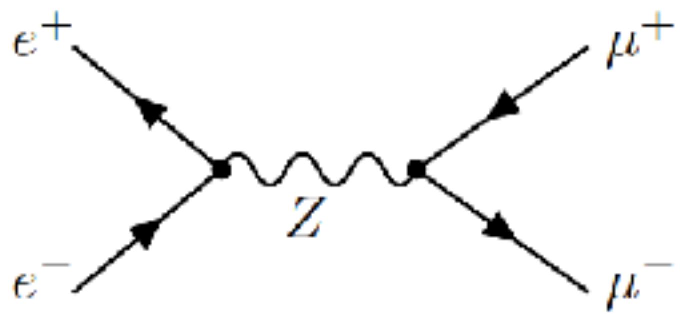
Intermezzo

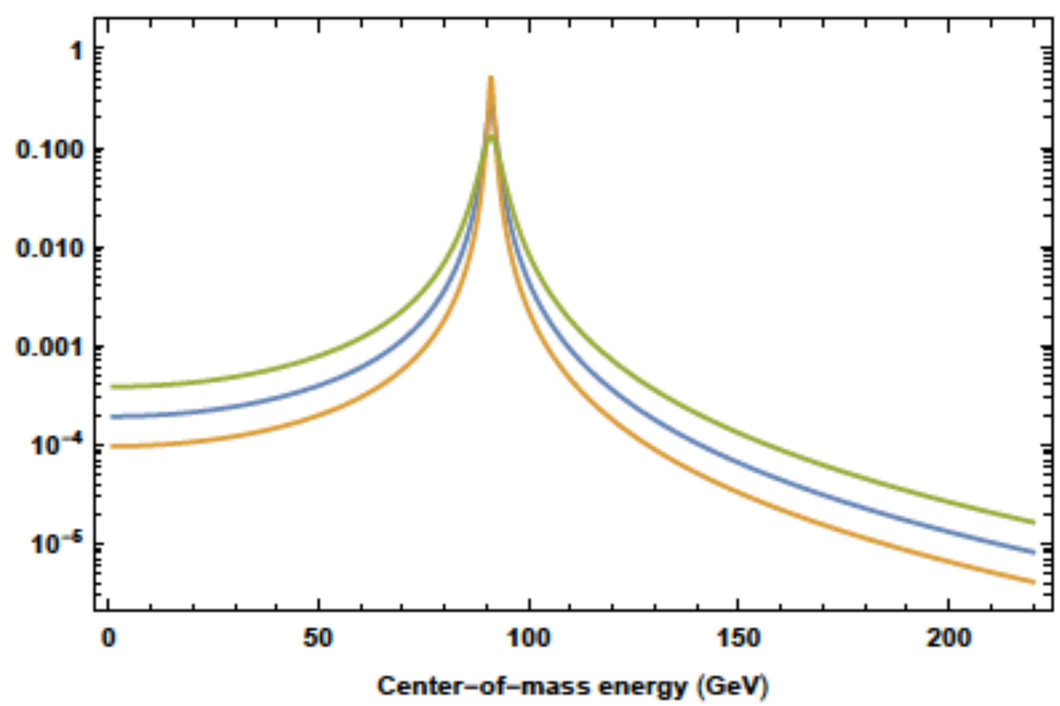


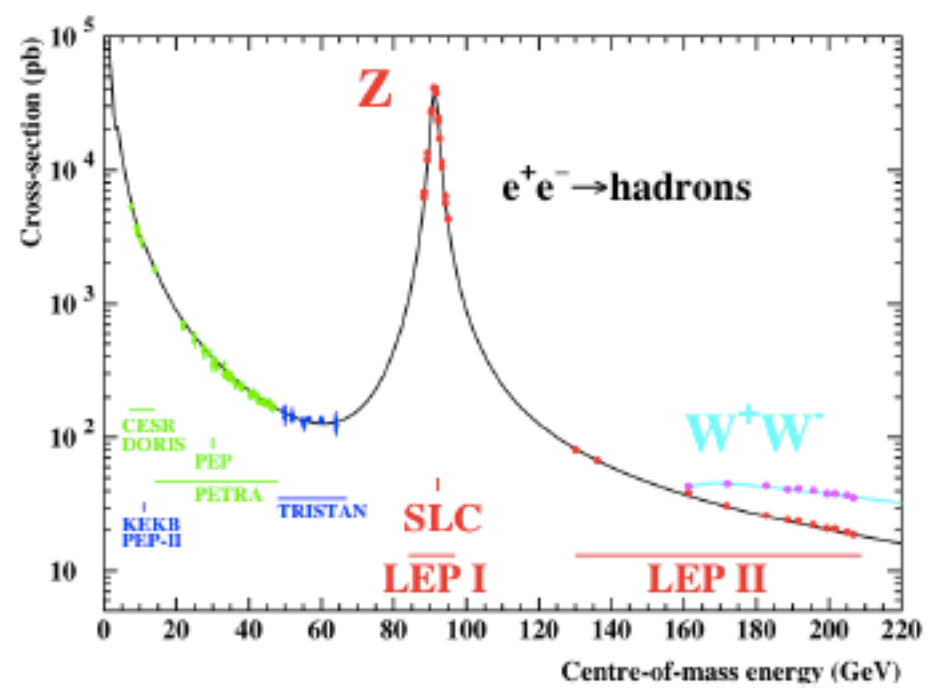
Wahrscheinlichkeit: $\left| e^{iS[P_1]} + e^{iS[P_2]} \right|^2$

Wirkungsquerschnitte

$E \rightarrow$







Wirkungsquerschnitte

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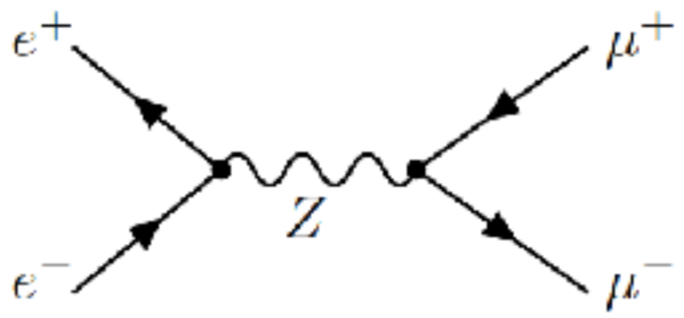
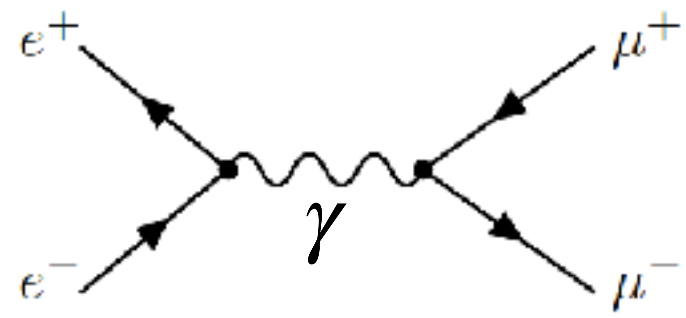
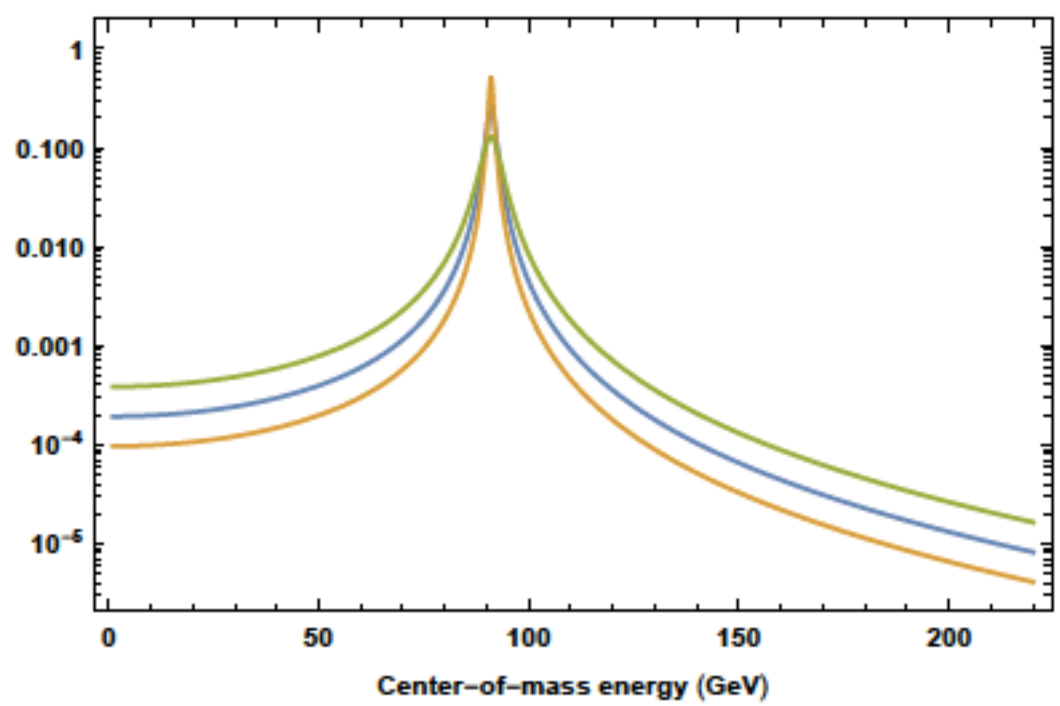
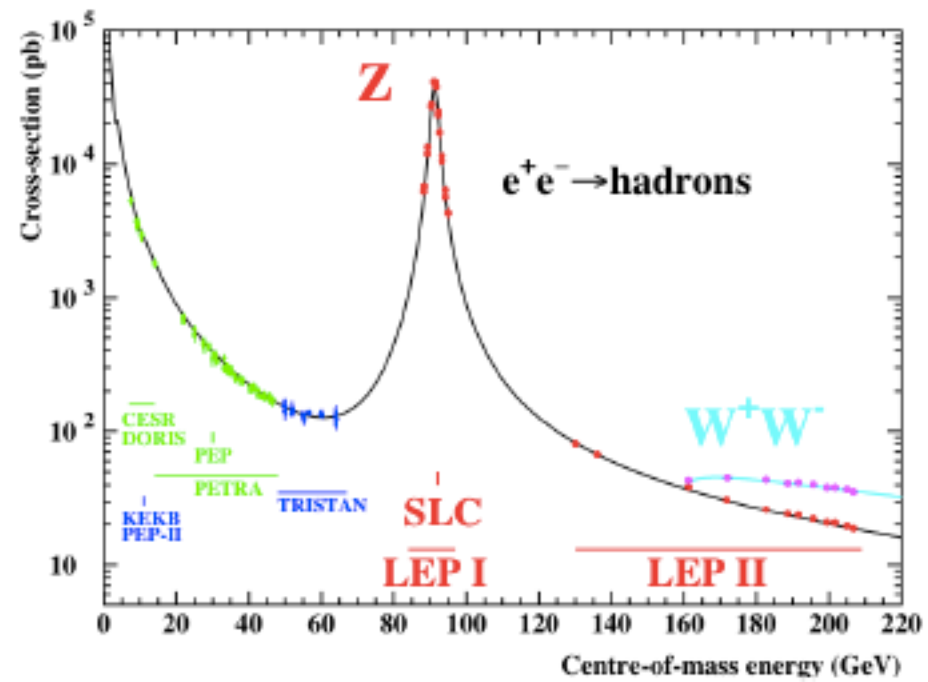


$+$







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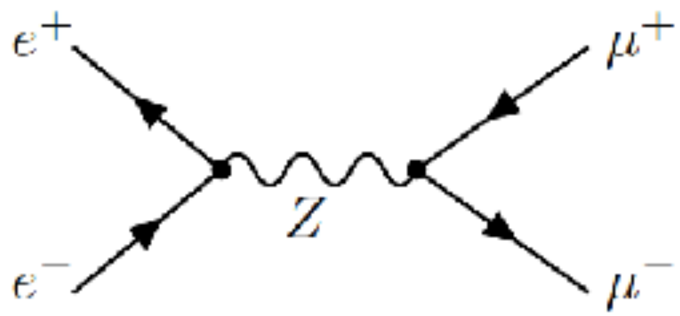
Wirkungsquerschnitte

$E \rightarrow$



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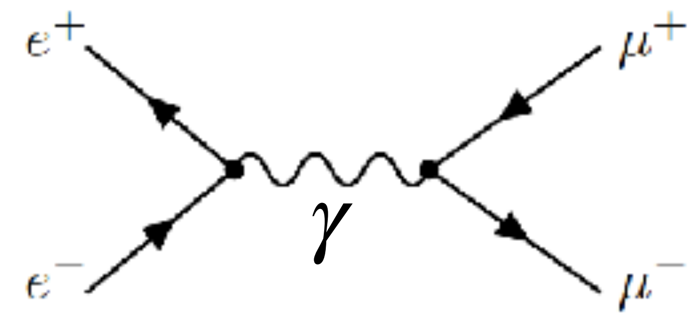
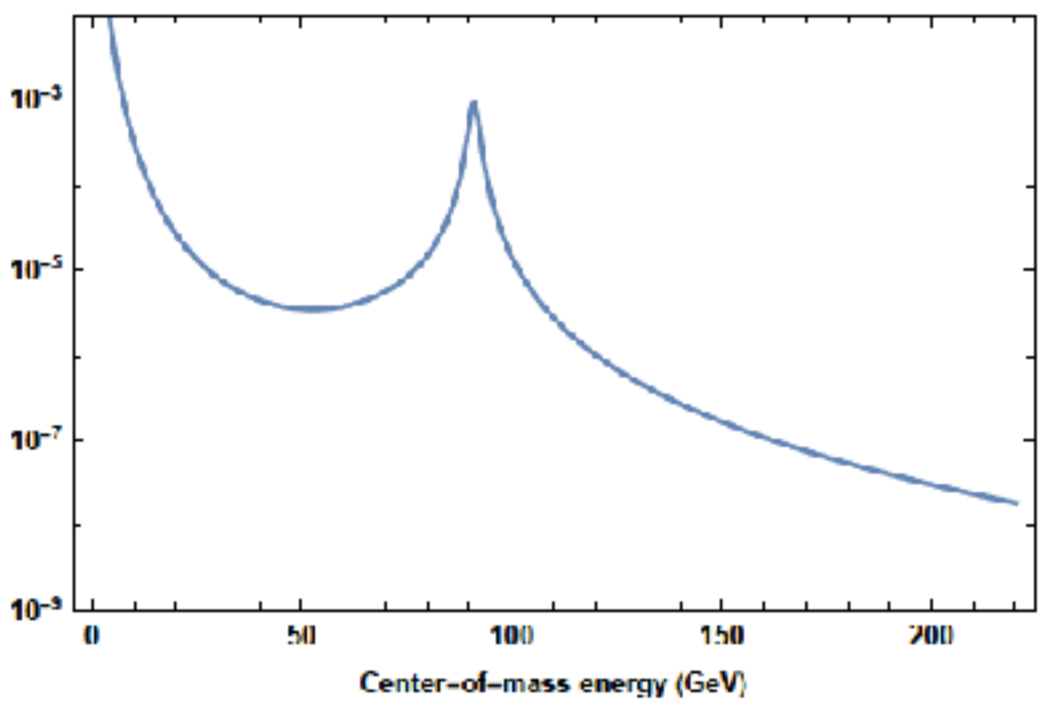
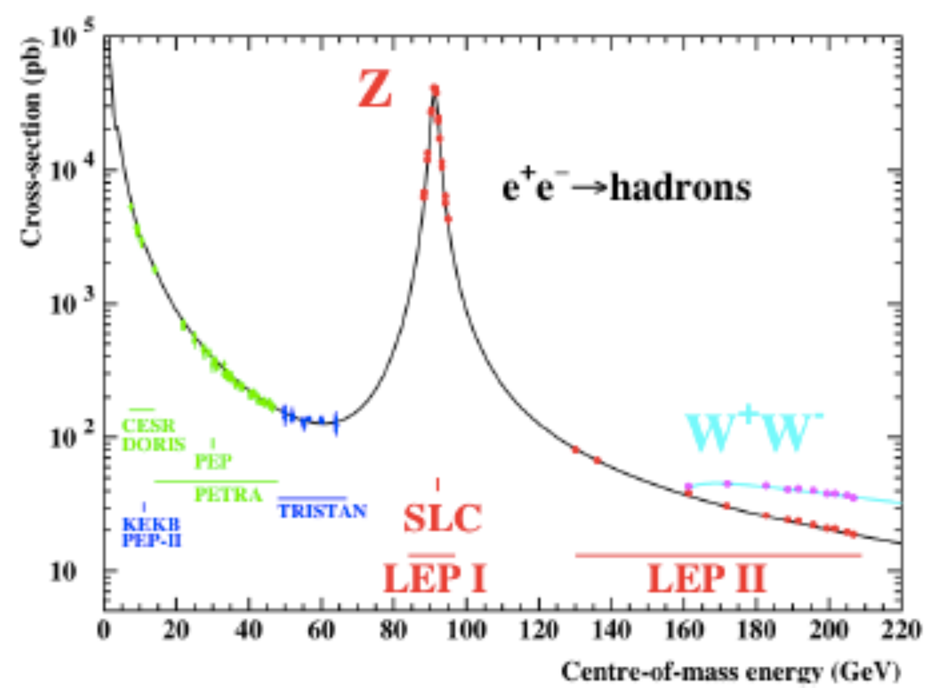
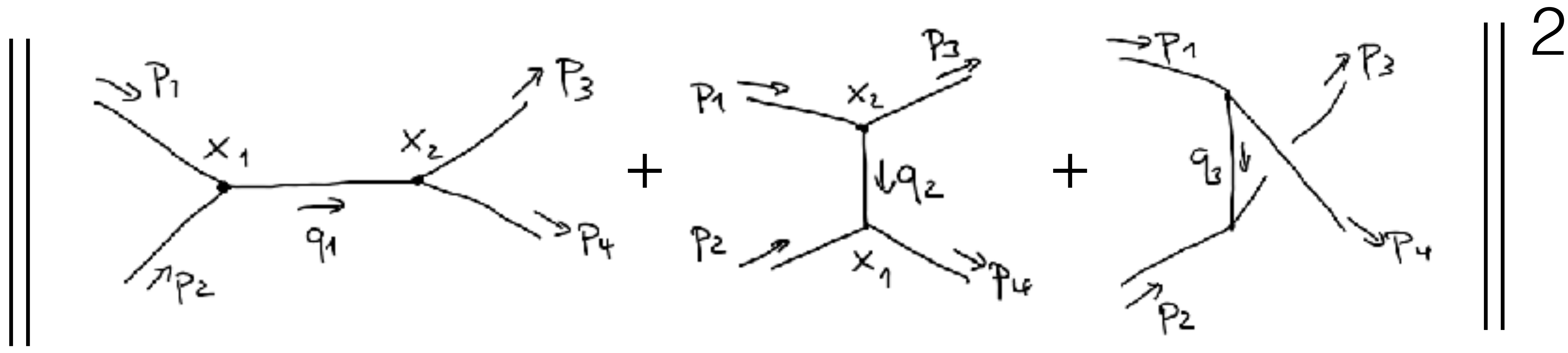






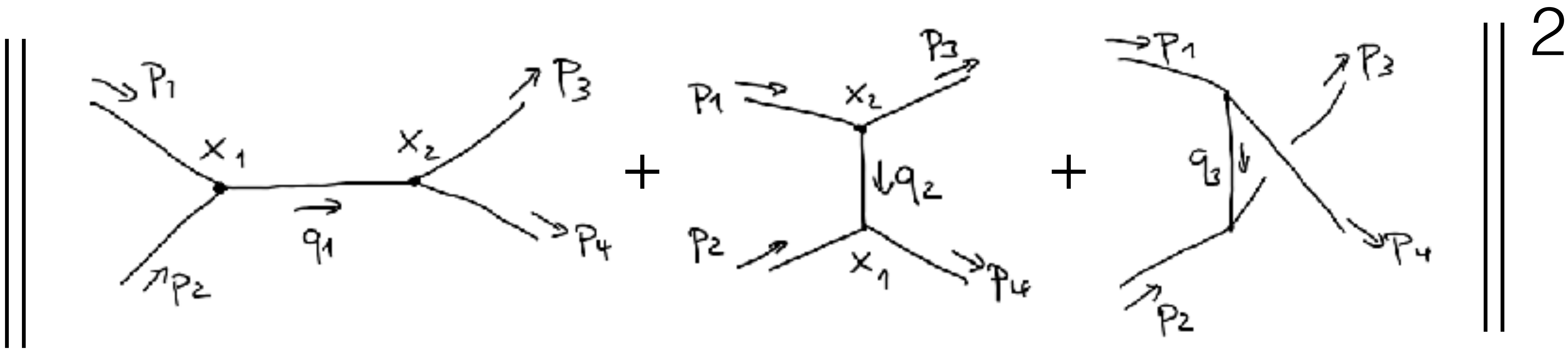
Wirkungsquerschnitte

Summe über Feynman-Diagramme, quadriert



Wirkungsquerschnitte

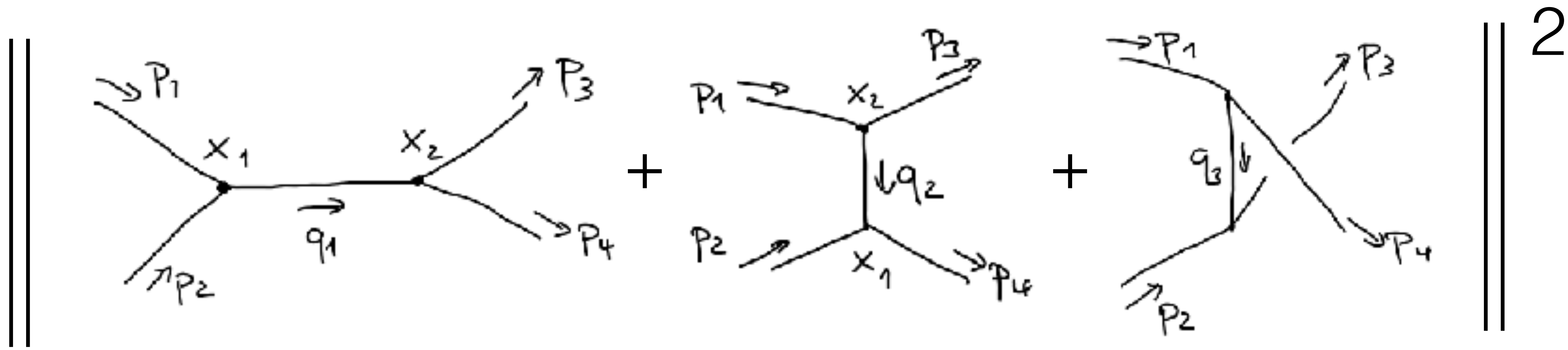
Summe über Feynman-Diagramme, quadriert



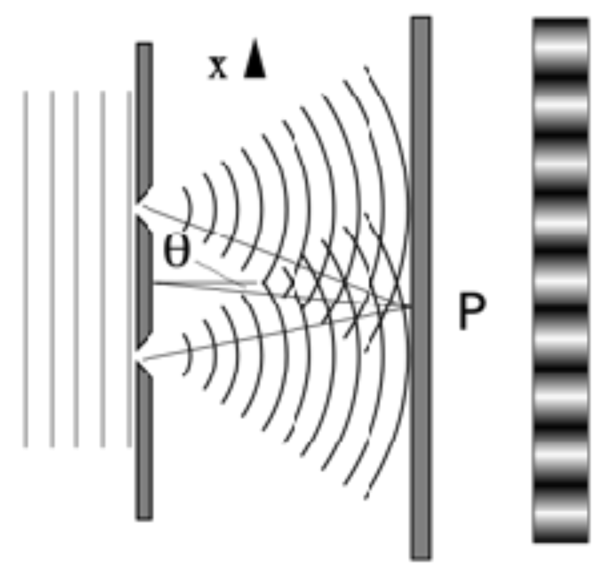
Interferenz-Effekte!

Wirkungsquerschnitte

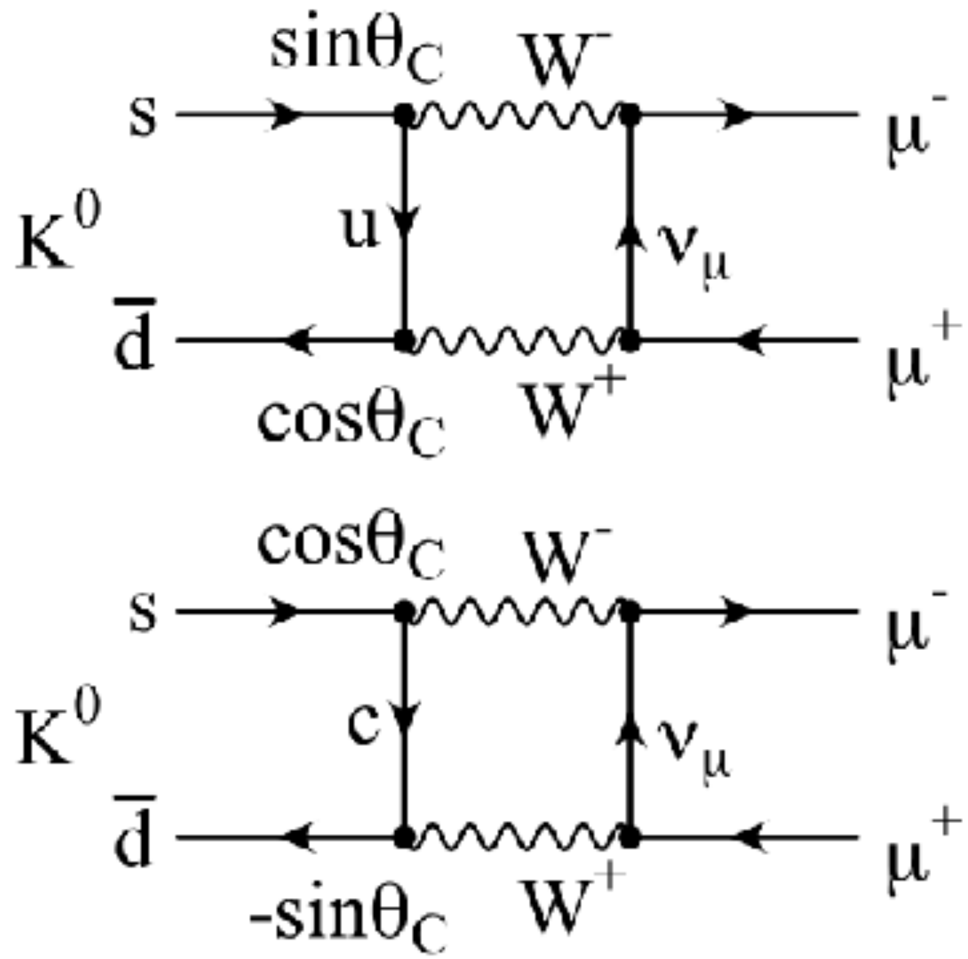
Summe über Feynman-Diagramme, quadriert



Interferenz-Effekte!



Interferenz-Effekte



Flavor-changing neutral currents

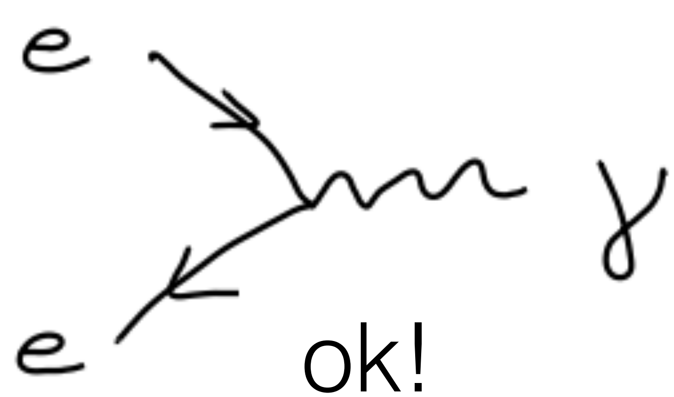
GIM-Mechanismus
(Glashow-Iliopoulos-Maiani)

→ destruktive Interferenz

Vorhersage des Charm-Quarks

Schleifen-Diagramme

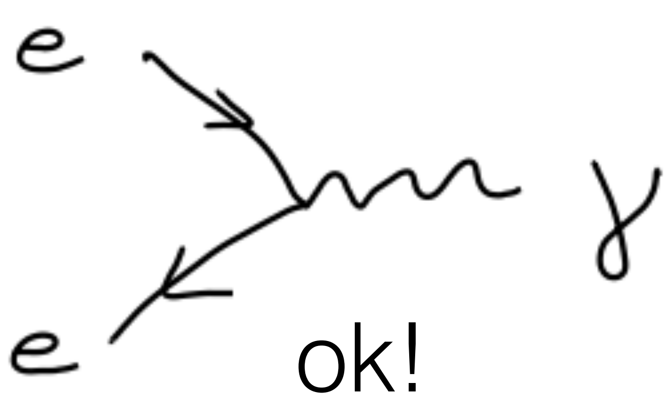
Photonen koppeln an elektrische Ladung



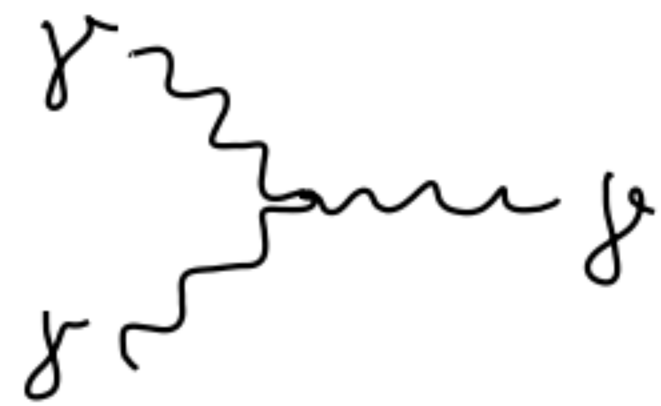
ok!

Schleifen-Diagramme

Photonen koppeln an elektrische Ladung

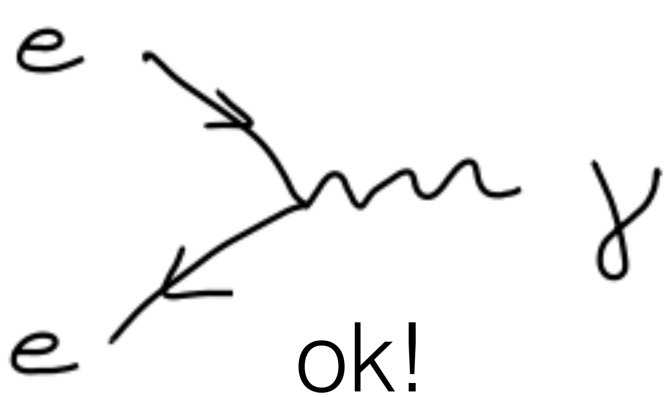


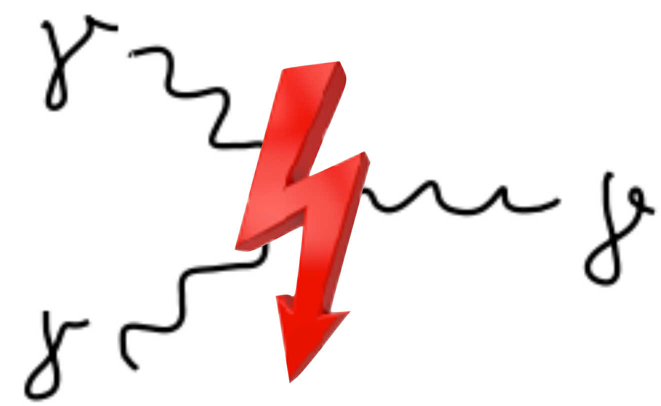
ok!



Schleifen-Diagramme

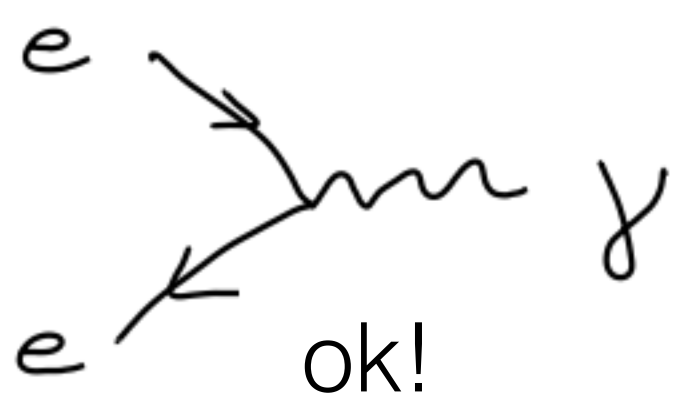
Photonen koppeln an elektrische Ladung

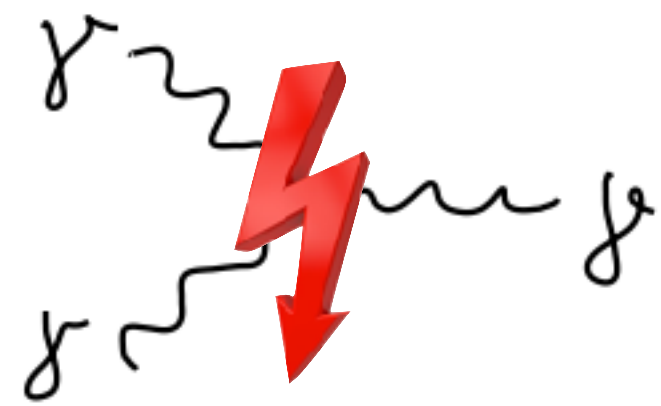




Schleifen-Diagramme

Photonen koppeln an elektrische Ladung





Licht-Licht-Streuung:
Schleifen-induziert!

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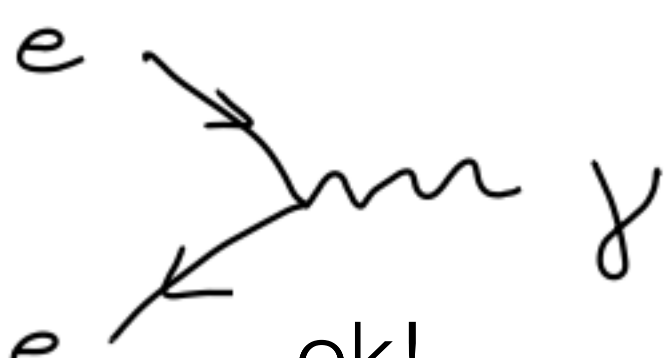
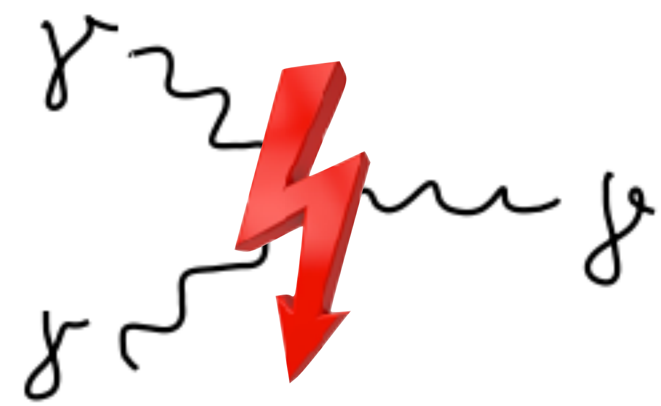
Schleifen-Diagramme

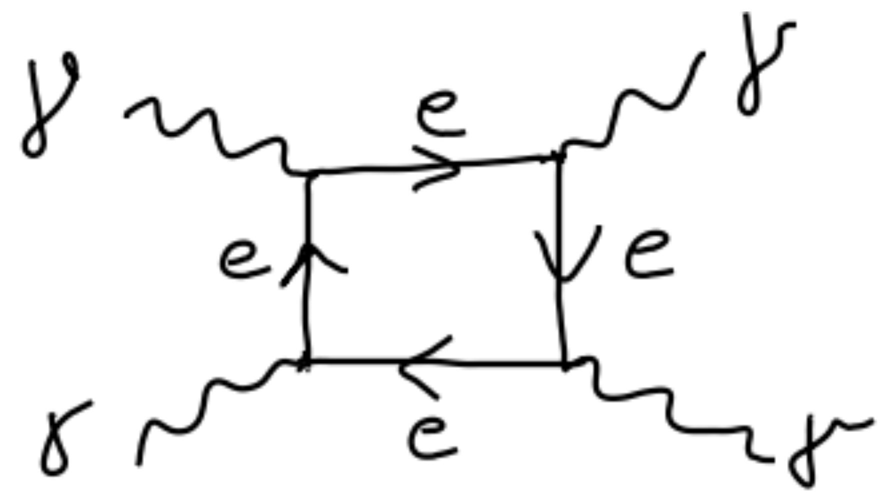
Photonen koppeln an elektrische Ladung



ok!

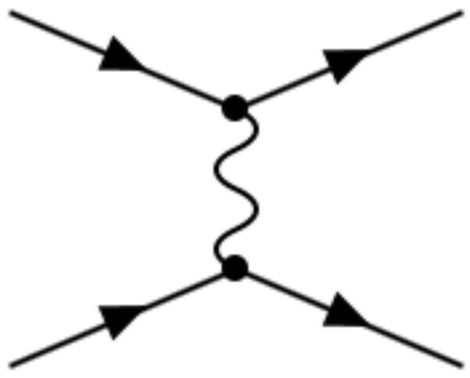


Licht-Licht-Streuung:
Schleifen-induziert!



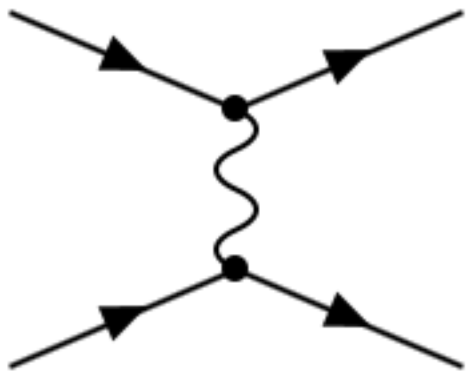
(Fehl?)Interpretation

Betrachte Elektron-Elektron-Streuung:



(Fehl?)Interpretation

Betrachte Elektron-Elektron-Streuung:



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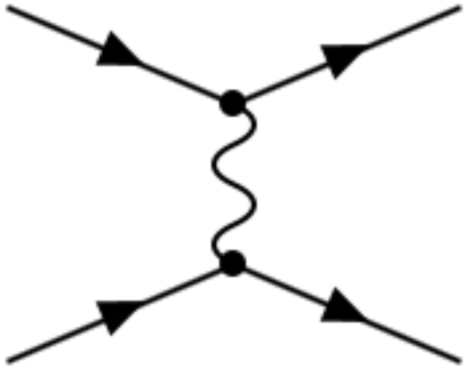
(Fehl?)Interpretation

Betrachte Elektron-Elektron-Streuung:



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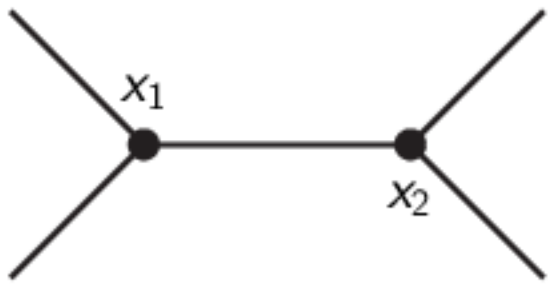
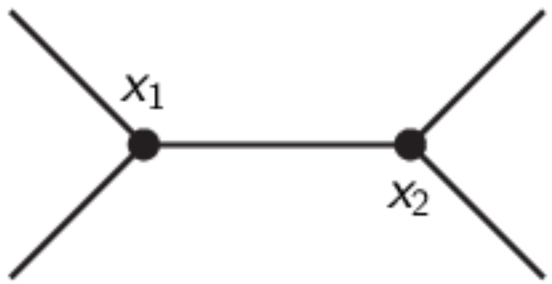
(Fehl?)Interpretation



(Fehl?)Interpretation



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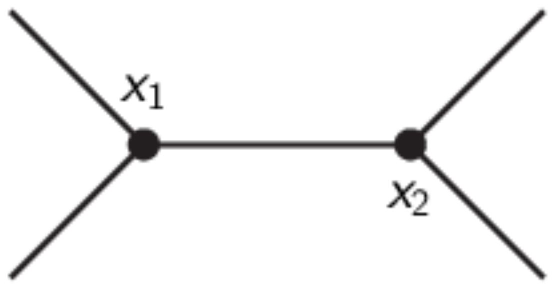

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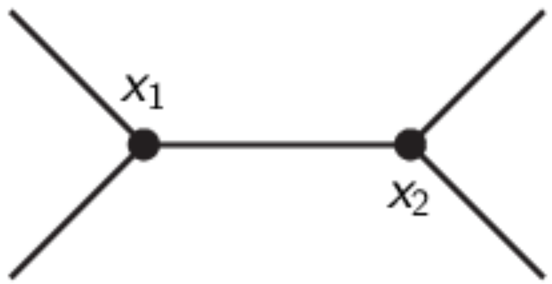
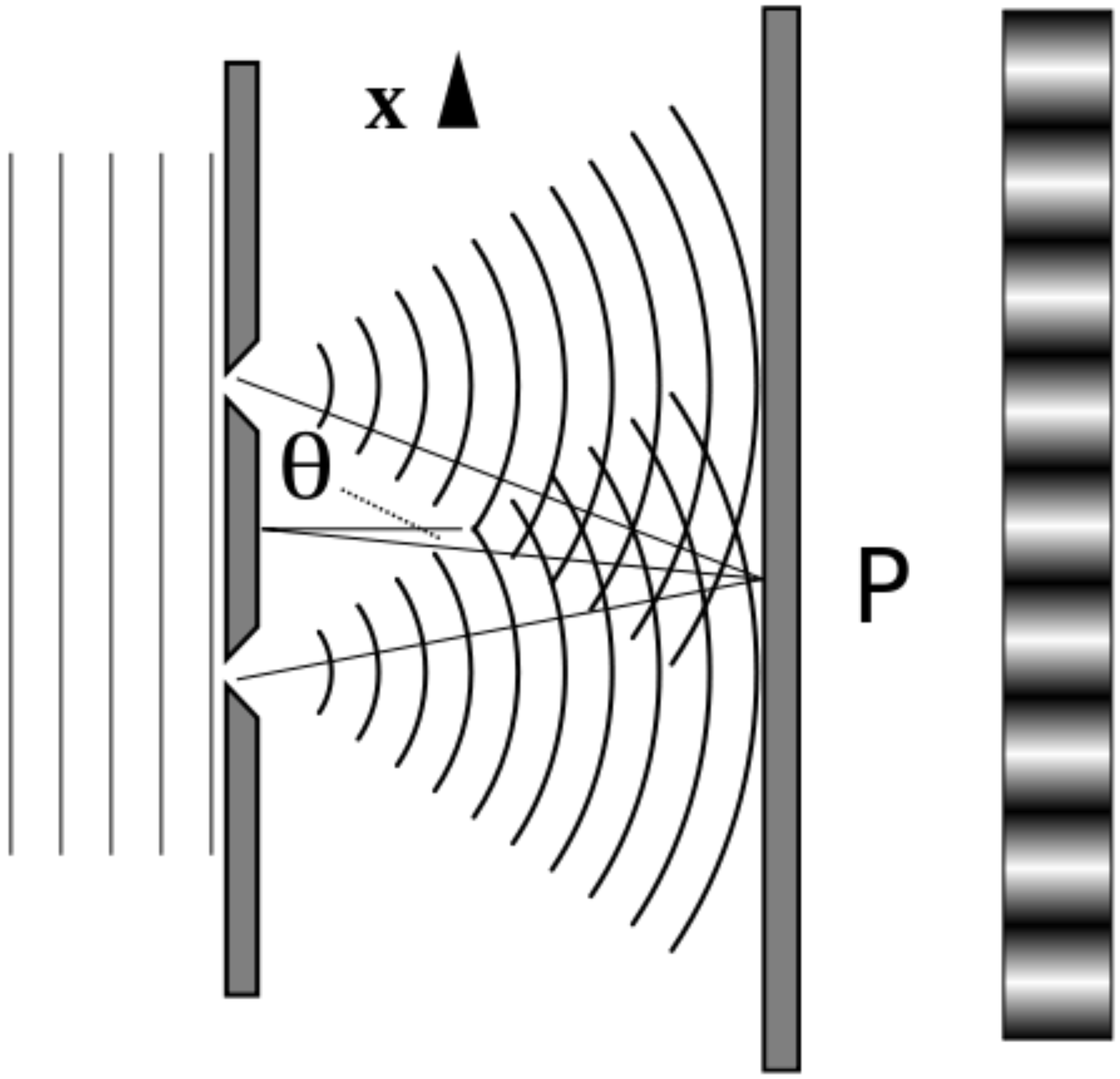
(Fehl?)Interpretation



Interpretation



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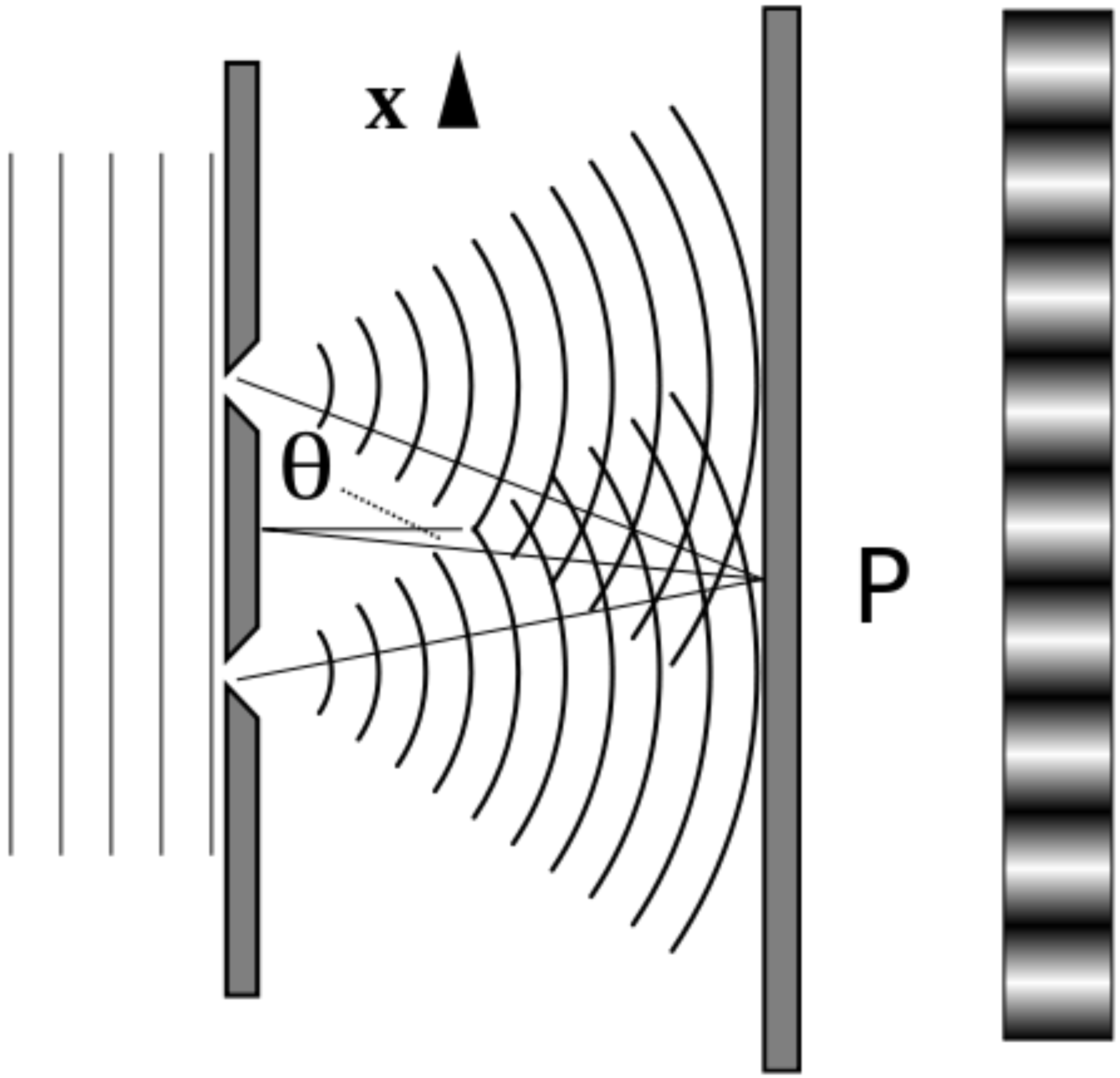

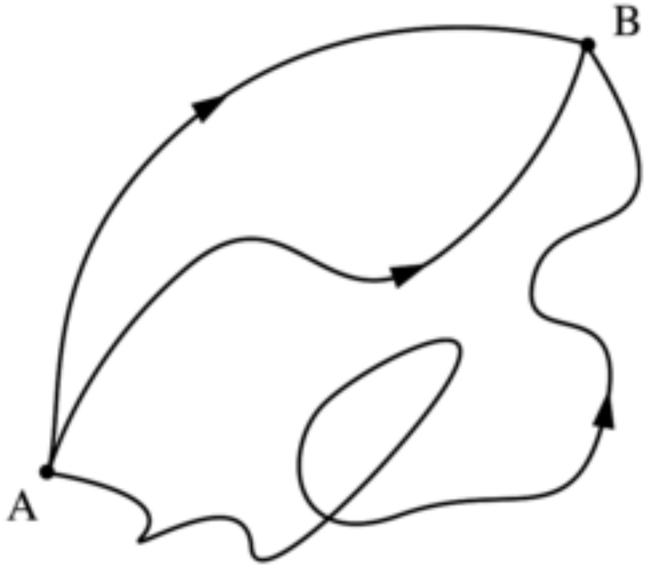

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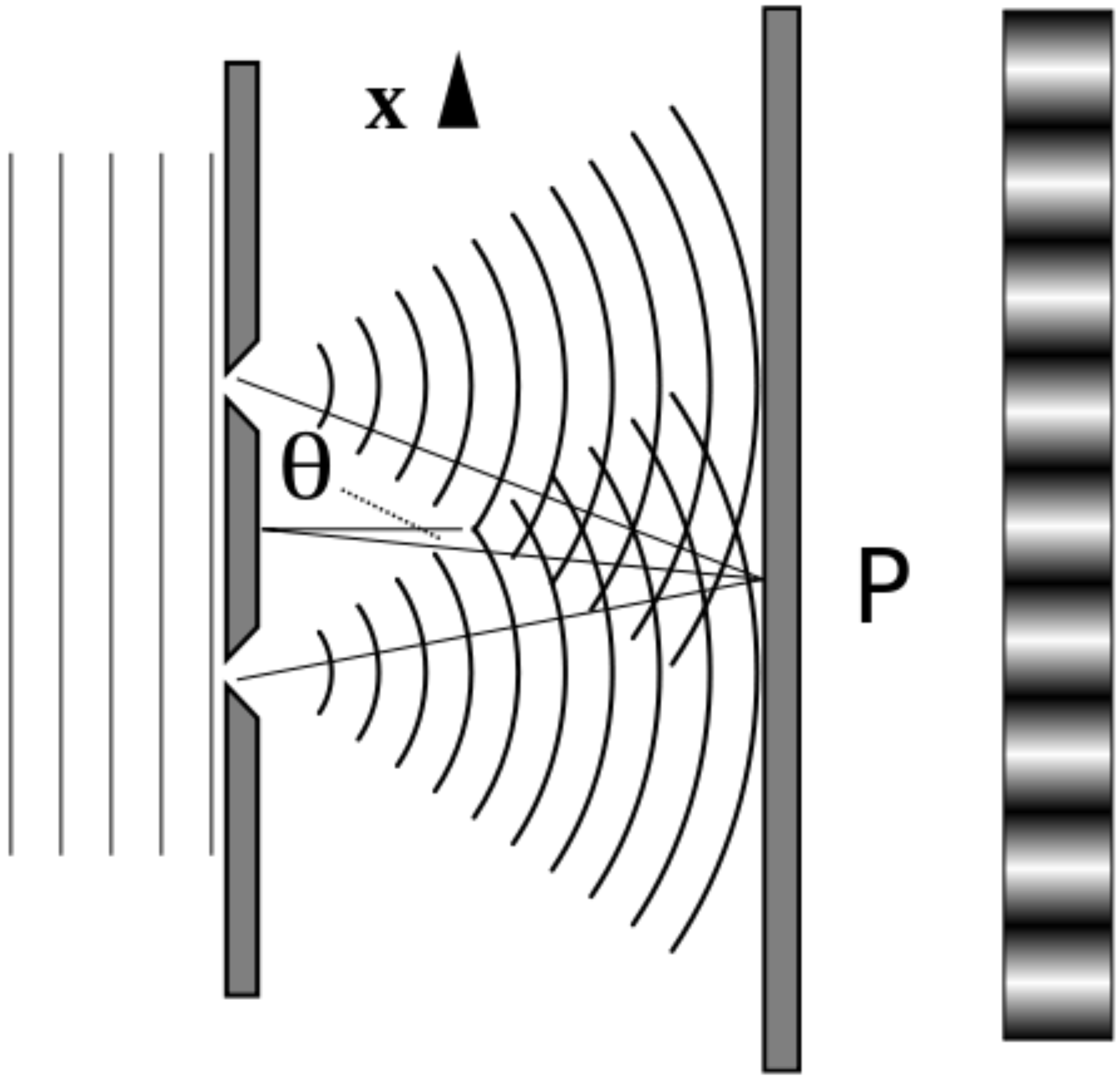
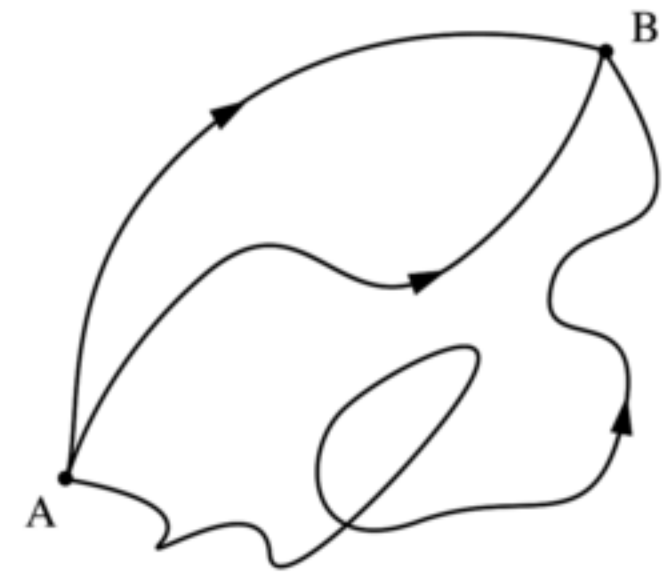
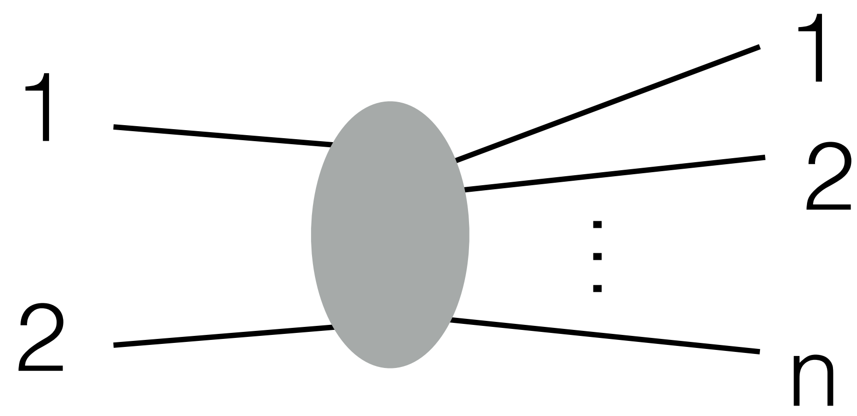




$$Z(B,A) = \mathcal{N}\int \mathcal{D}q\,\exp\left(\frac{\mathrm{i}}{\hbar}S\right)$$

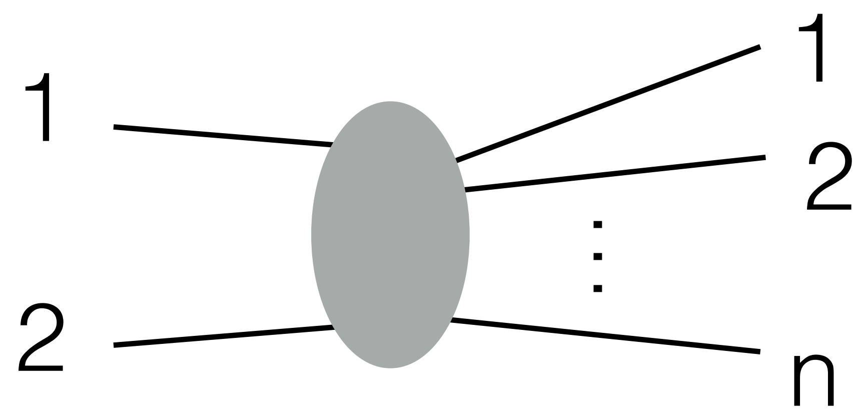
Pfadintegral

Formeln…



Formeln…

$$P_i = p_1 + p_2$$



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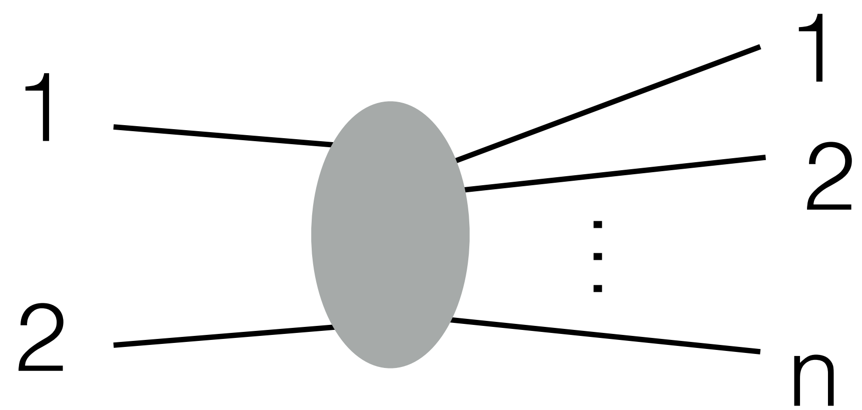
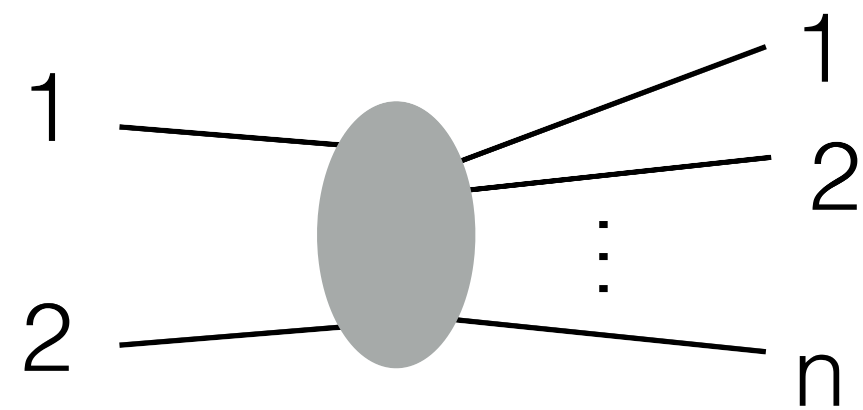


$$P_f = p_1' + p_2' + \cdots + p_n'$$

Formeln…

$$P_i = p_1 + p_2$$



$$P_f = p_1' + p_2' + \cdots + p_n'$$

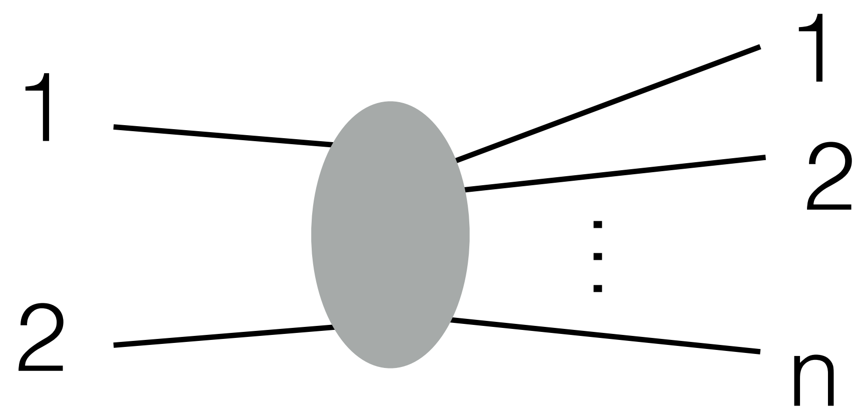
Wirkungsquerschnitt:

$$d\sigma = \frac{1}{4E_1E_2|\vec{v}_1-\vec{v}_2|}\,|\mathcal{M}_{fi}|^2\, d\Pi_{LIPS}$$

$$d\Pi_{LIPS} = (2\pi)^4\,\delta(P_f - P_i)\prod_{j=3}^{n}\frac{d^3p_j}{(2\pi)^3\,2E_j}$$

Formeln…

$$P_i = p_1 + p_2$$



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$$d\Pi_{LIPS} = (2\pi)^4\,\delta(P_f-P_i)\prod_{j=3}^{n}\frac{d^3p_j}{(2\pi)^3\,2E_j}$$

$$S = \alpha\,\mathbb{1} + iT\,,$$

$$\langle f|T|i\rangle = (2\pi)^4\,\delta(P_f-P_i)\,\langle f|\mathcal{M}|i\rangle \quad \leftarrow \mathcal{M}_{fi}$$

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… mehr Formeln …

$$\langle f|S|i\rangle = 2^{n/2}\sqrt{E_1\cdots E_n}\,\lim_{T_i\to\infty}\langle\Omega|\underbrace{a_n a_{n-1}\cdots a^+_{-1}a^+_{-2}}_{\text{already time-ordered} \; = T(\cdots)}|\Omega\rangle$$

$$S|i\rangle = |i\rangle_{in}$$

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… mehr Formeln …

$$\langle f|S|i\rangle = 2^{n/2}\sqrt{E_1\cdots E_n}\lim_{T_i\to\infty}\langle\Omega|\underbrace{a_n a_{n-1}\cdots a^\dagger_{-1}a^\dagger_{-2}}_{\text{already time-ordered} \;=\; T(\cdots)}|\Omega\rangle$$

$$\underbrace{|i\rangle}_{=|i\rangle_{in}}$$

$$=\left[i\int d^4x_1\, e^{-ip_1x_1}(\Box_1+m^2)\right]\cdots\left[i\int d^4x_n\, e^{+ip_n\cdot x_n}(\Box_n+m^2)\right]$$

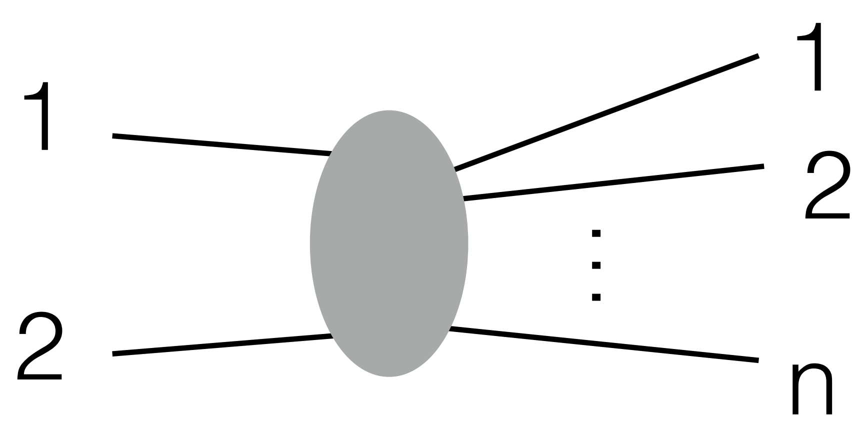
$$\times\langle\Omega|T\phi(x_1)\cdots\phi(x_n)|\Omega\rangle$$

LSZ reduction formula

… noch mehr Formeln …

$$\langle \Omega | T \phi(x_1) \cdots \phi(x_n) | \Omega \rangle =$$

$$= \boxed{\frac{\langle 0 | T \phi_0(x_1) \cdots \phi_0(x_n) \exp\left(i \int d^4x \, \mathcal{L}_{int}(\phi_0)\right) | 0 \rangle}{\langle 0 | T \exp\left(i \int d^4x \, \mathcal{L}_{int}(\phi_0)\right) | 0 \rangle}}$$



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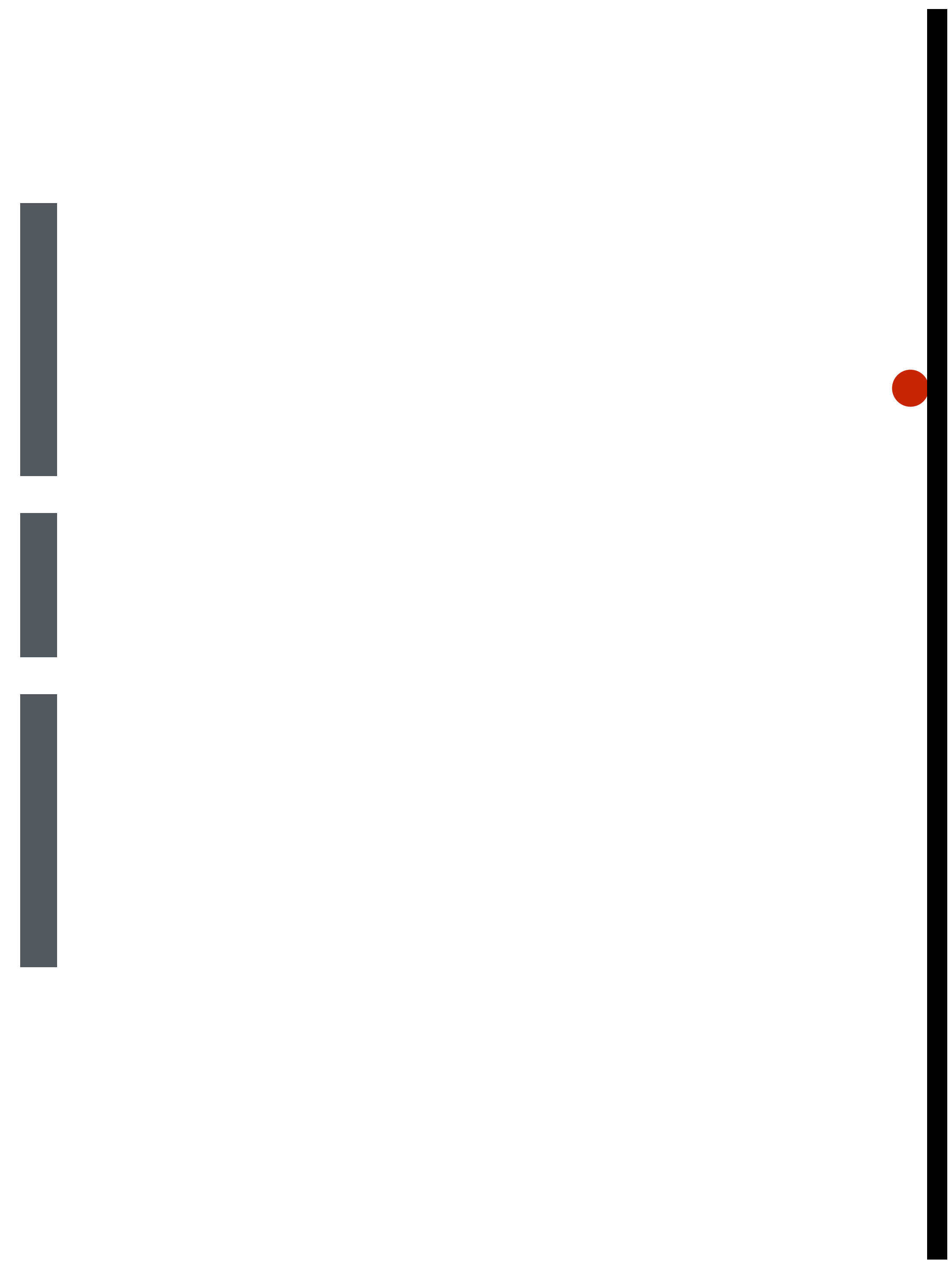
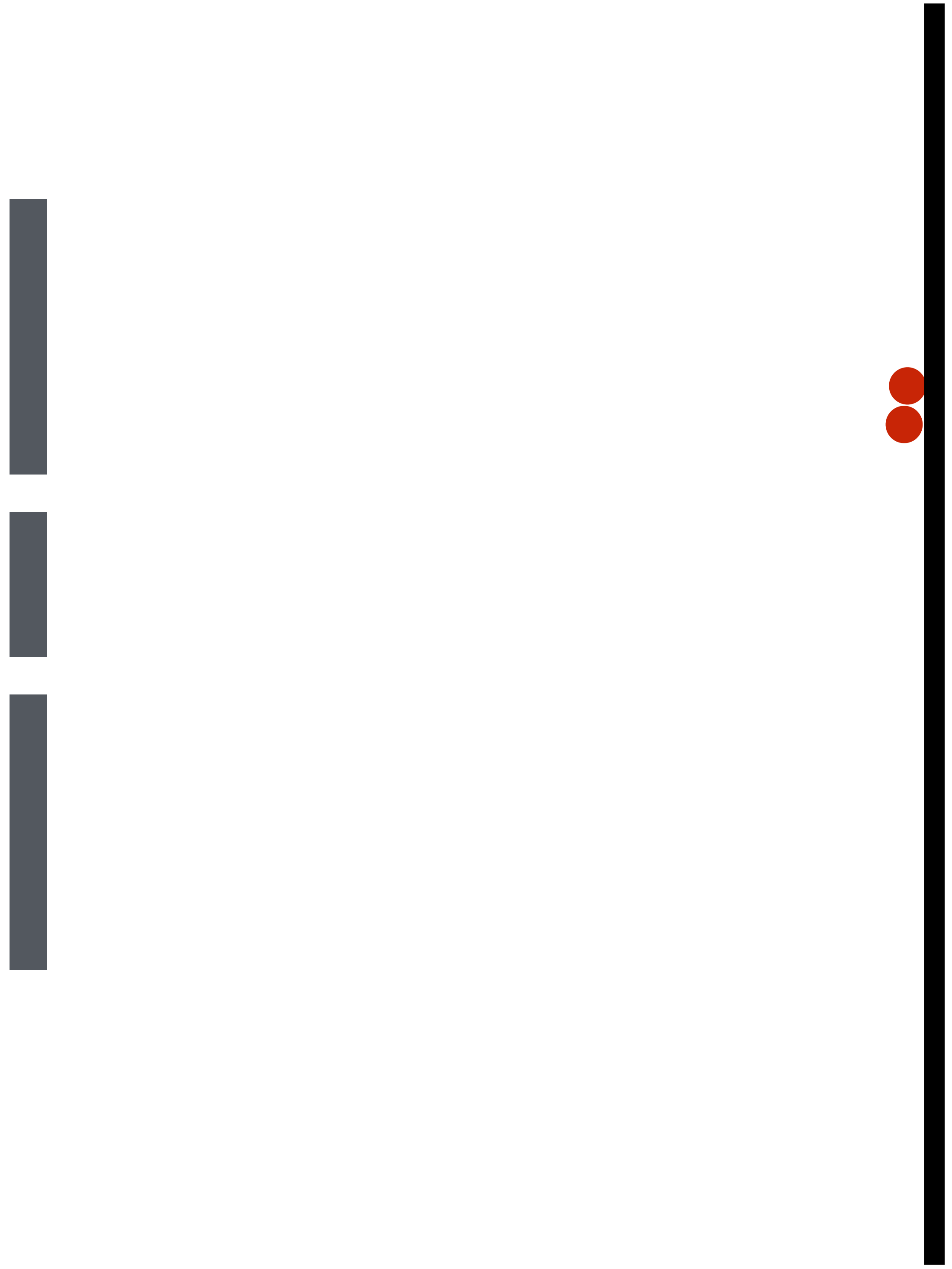
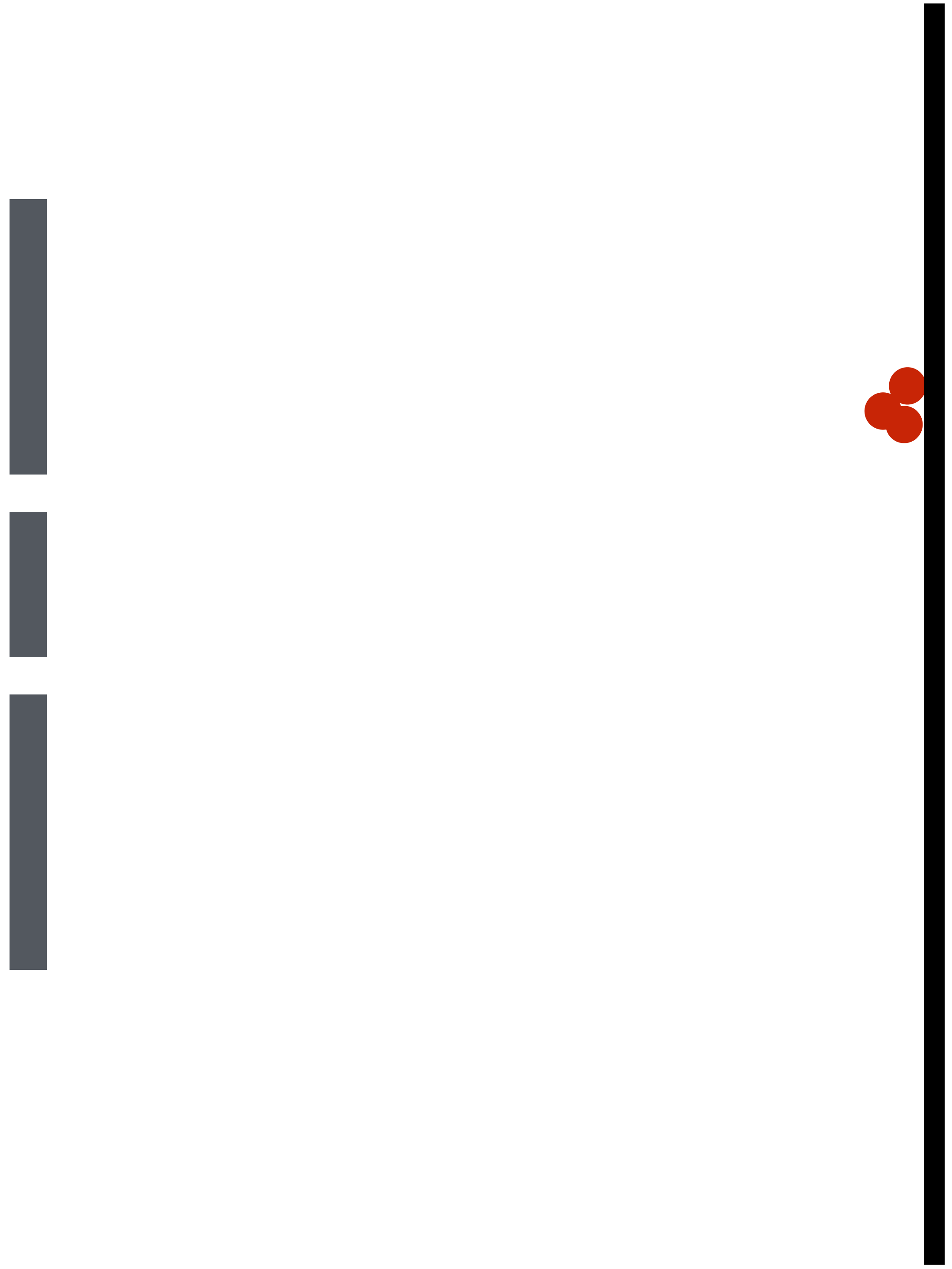
aximate

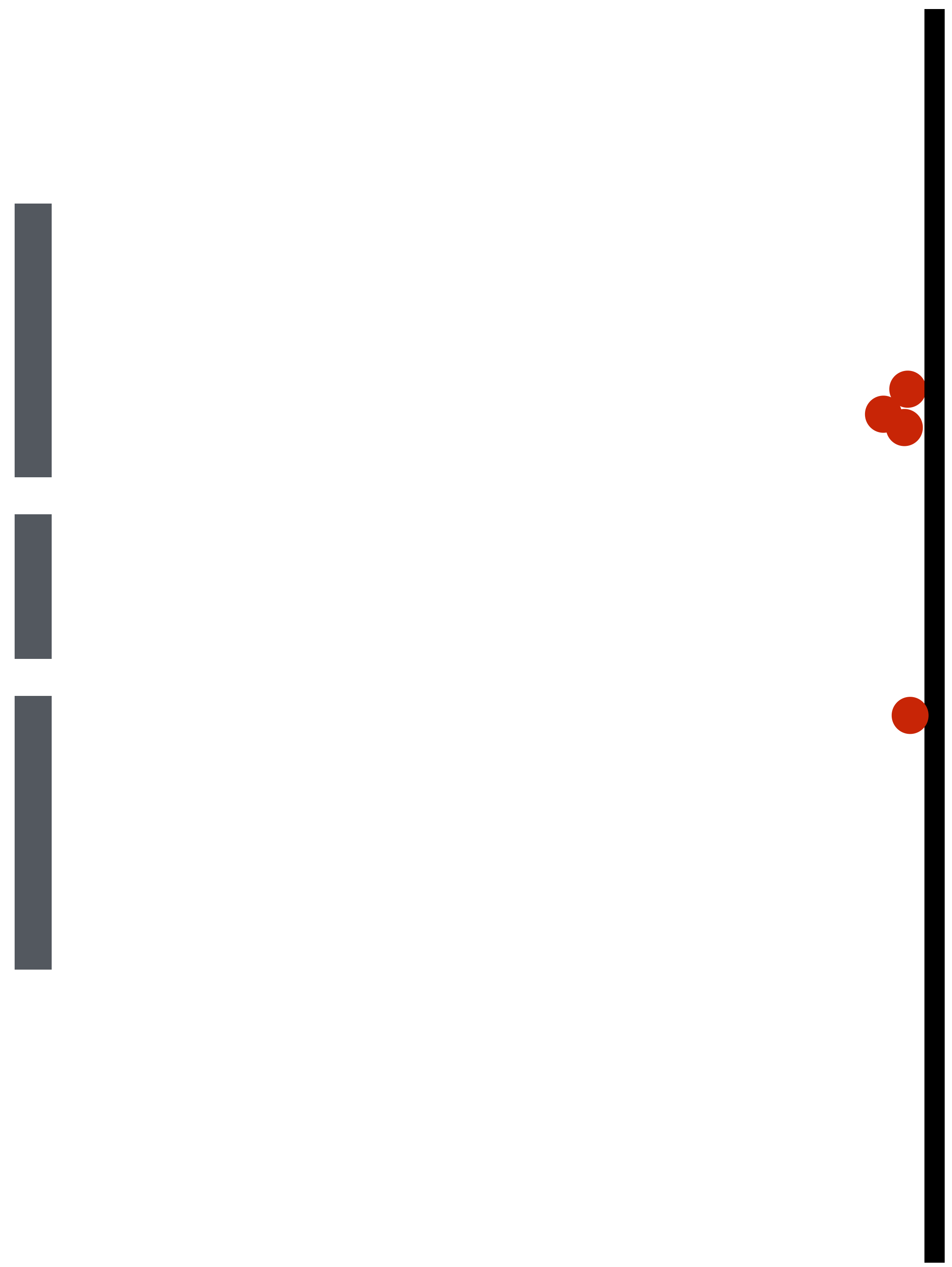
aximate

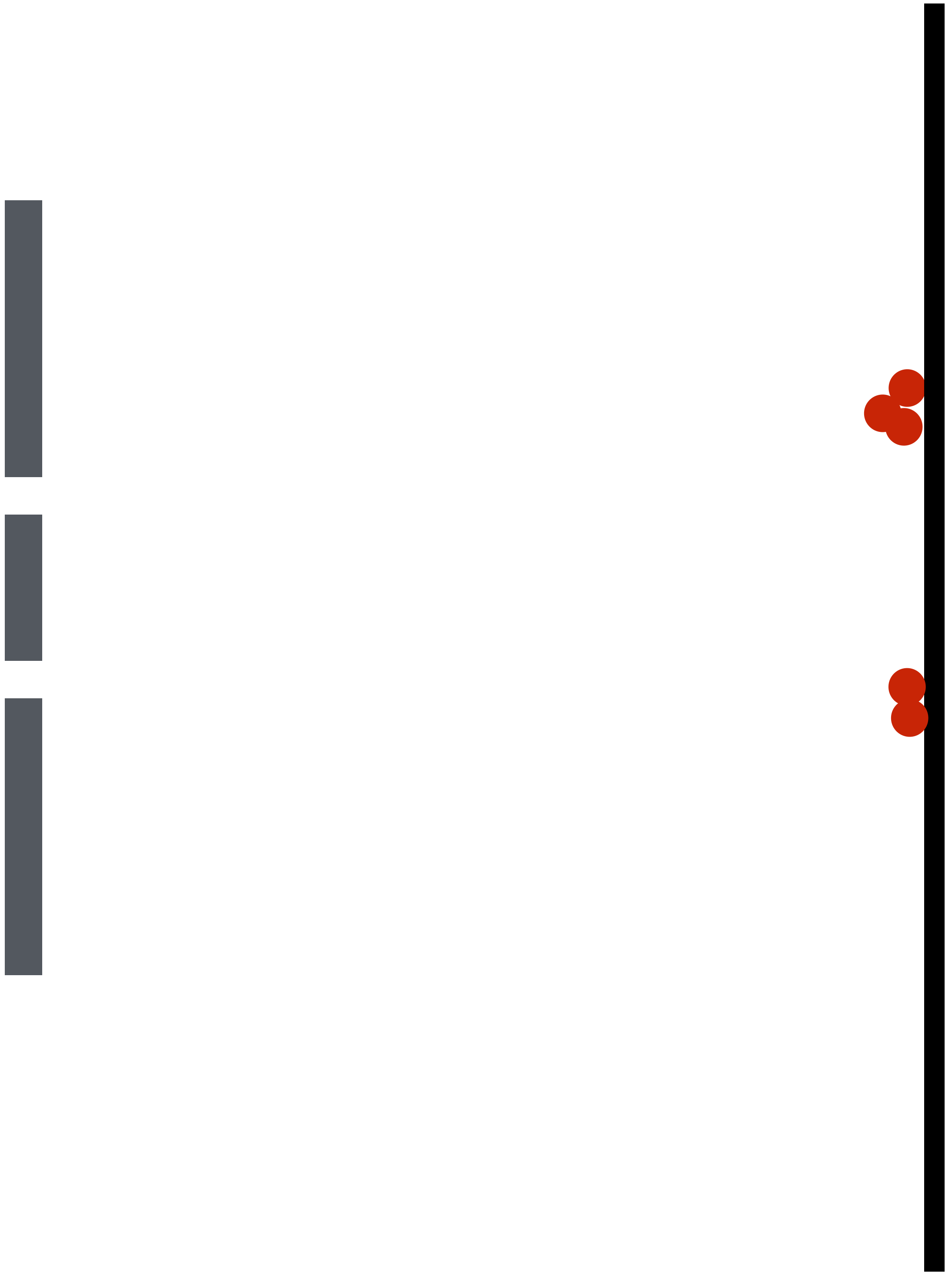
http://www.robert-harlander.de

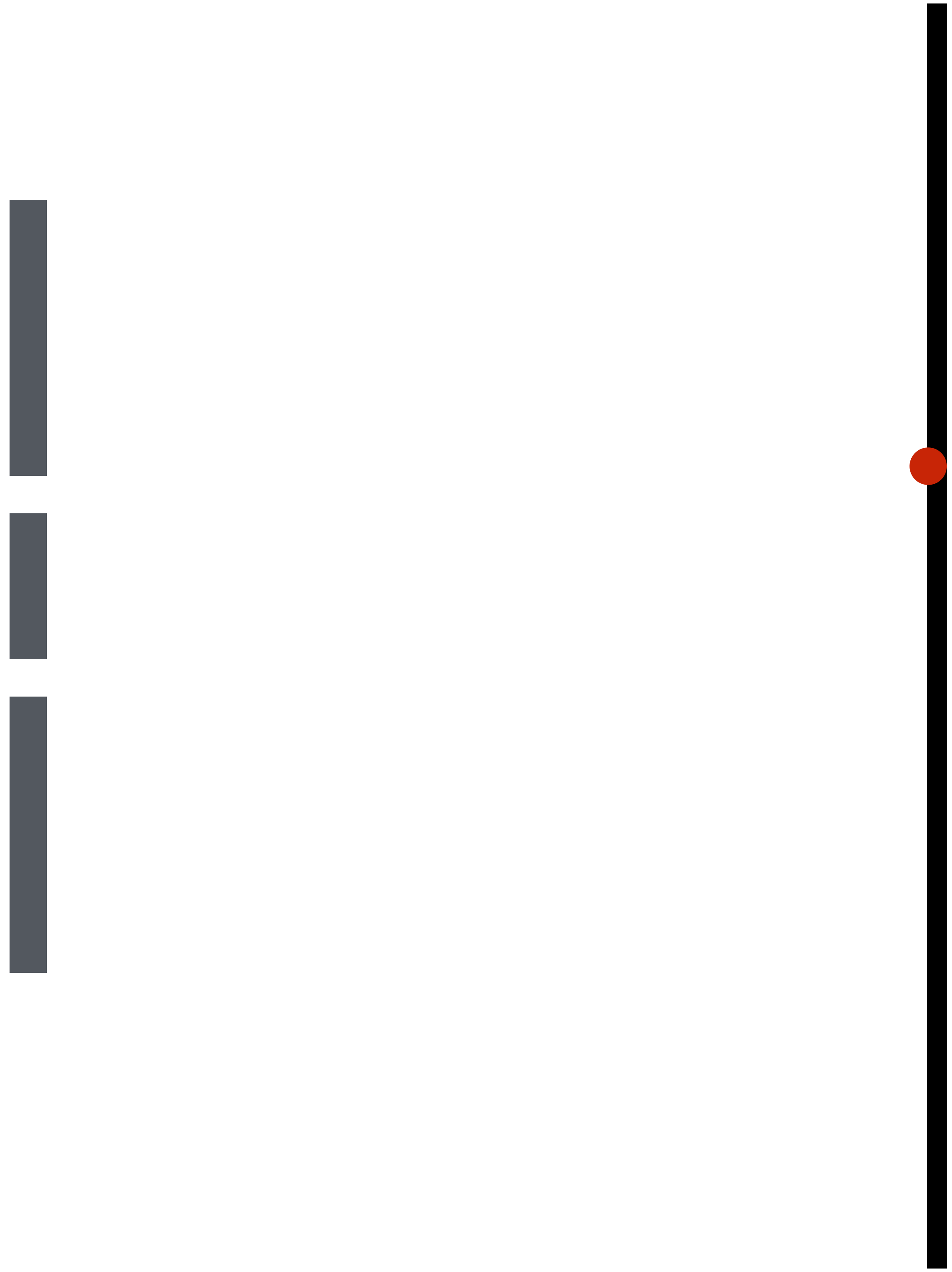


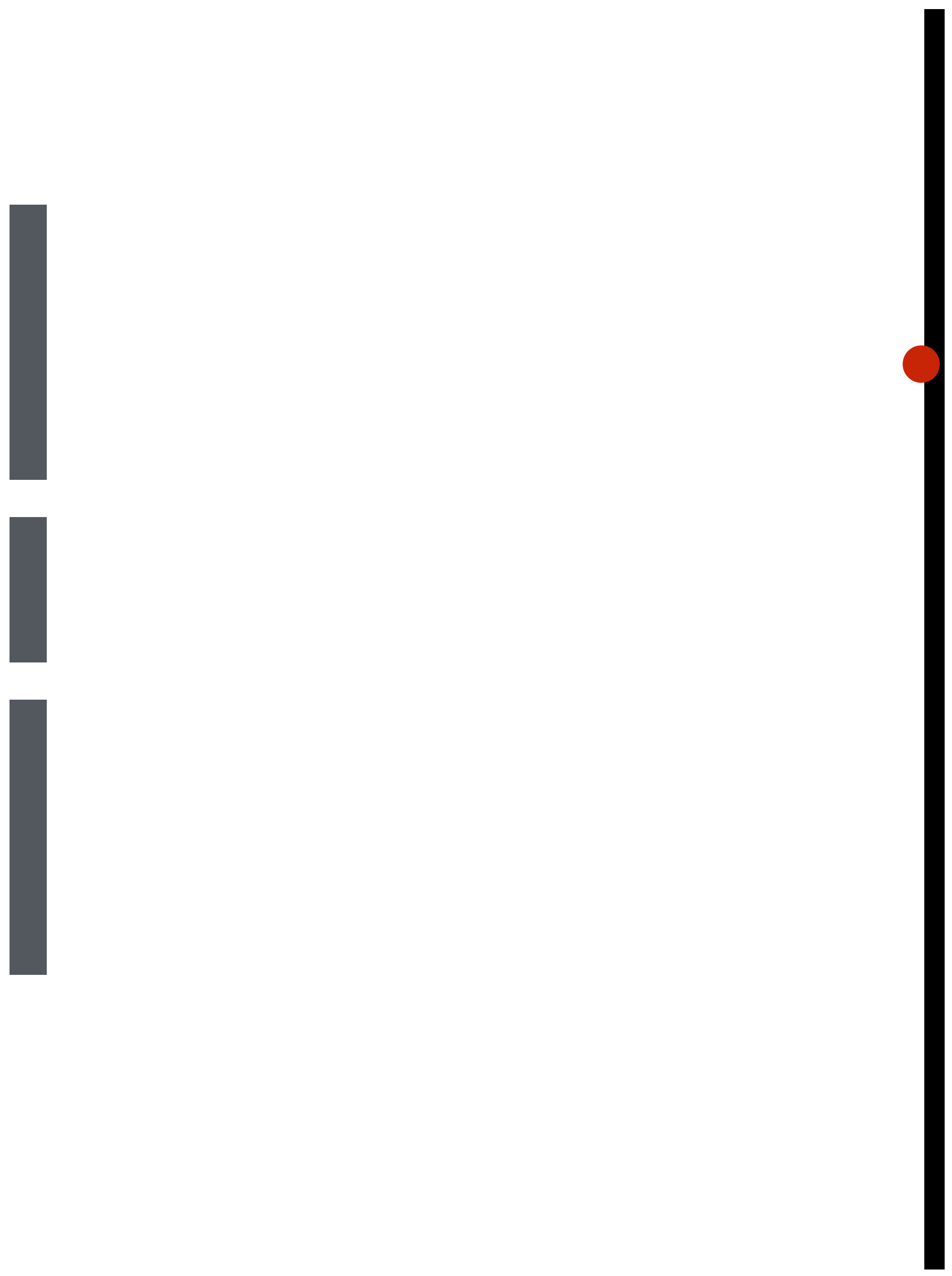


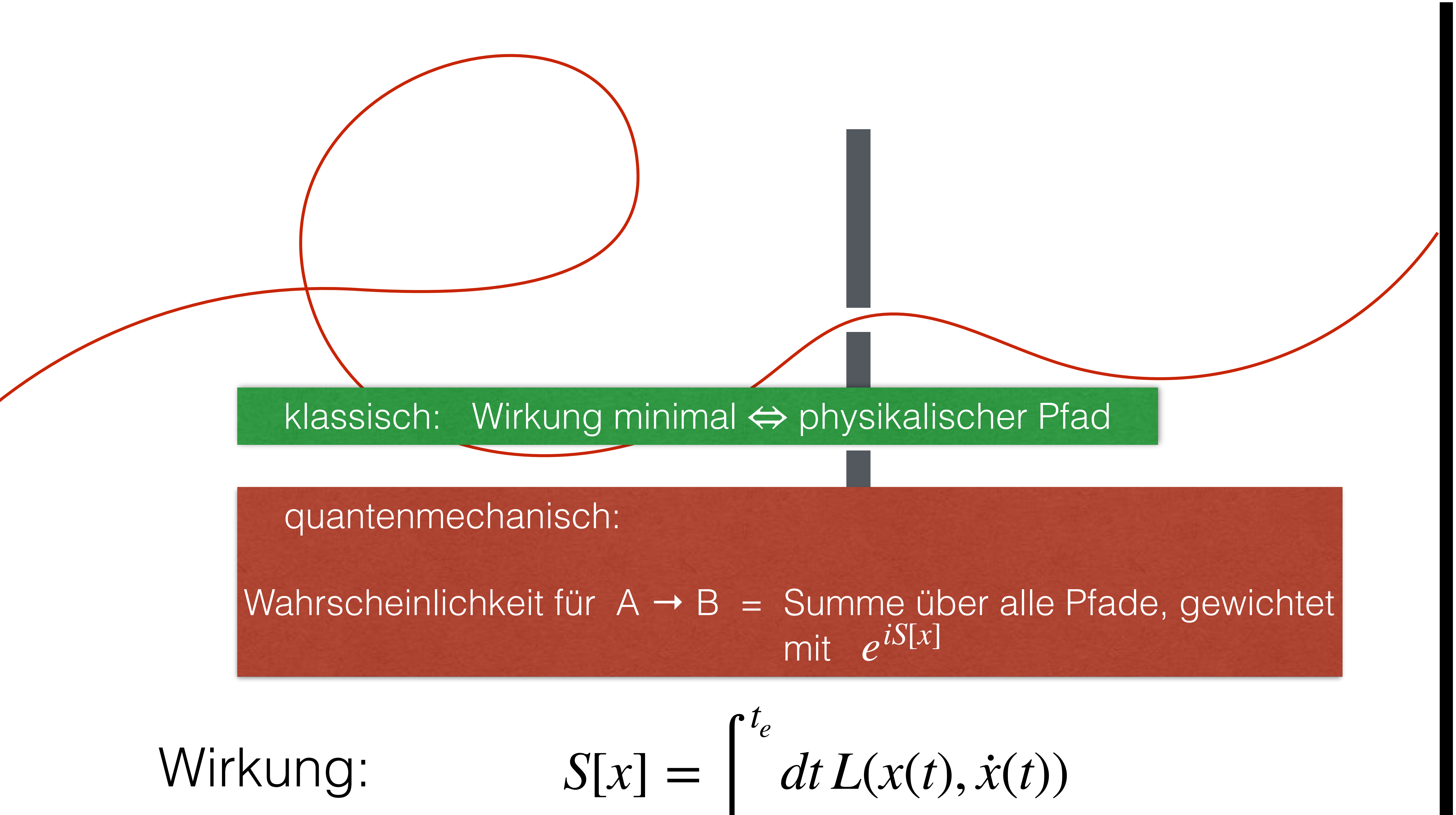










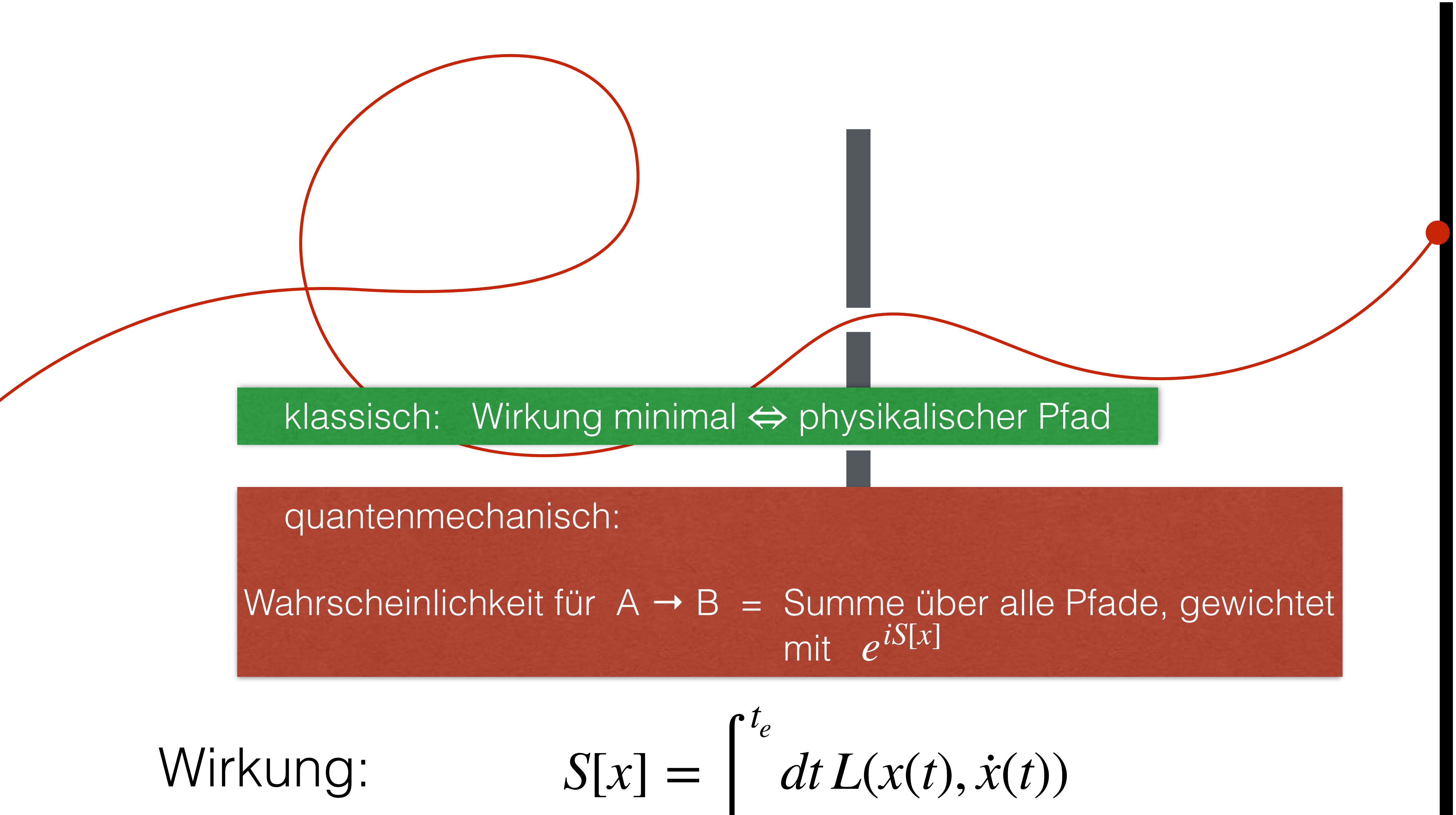


klassisch: Wirkung minimal ⇔ physikalischer Pfad

quantenmechanisch:

Wahrscheinlichkeit für A → B = Summe über alle Pfade, gewichtet mit $e^{iS[x]}$

Wirkung: $$S[x] = \int_{t_a}^{t_e} dt\, L(x(t), \dot{x}(t))$$



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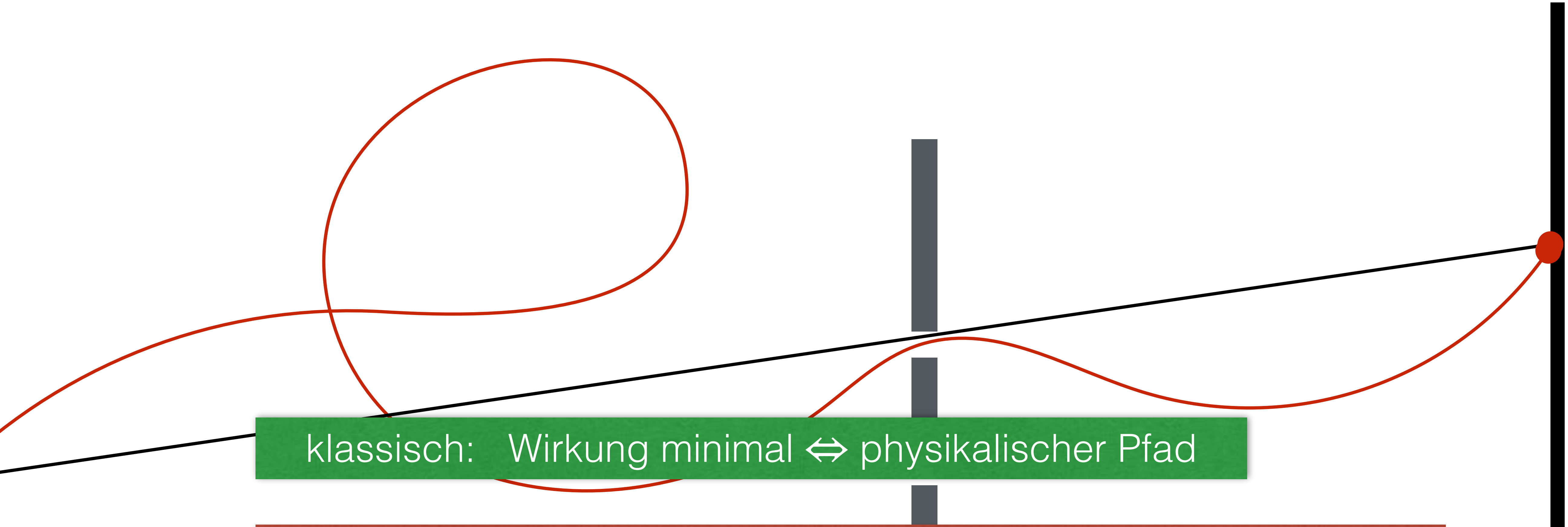

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$$W(A \to B) \sim \sum_{\text{Pfade}} e^{iS[x]} = \int \mathscr{D}x \, e^{iS[x]}$$